THE
STRESS-DEFORMATION CHARACTERISTICS
OF ASPHALTIC MIXTURES
UNDER
VARIOUS CONDITIONS OF LOADING

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by

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PURDUE UNIVERSITY
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TO: K. B. Woods, Director
       Joint Highway Research Project   September 26, 1956

FROM: Harold L. Michael, Assistant Director   File: 2-5-5-10
                                                C-36-6J

      The attached report entitled "The Stress-Deformation Character-
istics of Asphaltic Mixtures Under Various Conditions of Loading"
covers work done by Dr. J. E. Mcod in the bituminous laboratory for
developing relationships which were considered fundamental in
understanding bituminous mixture behavior particularly when used as
a portland cement concrete overlay.

      The data in this report are based, for the most part, on sheet
asphalt mixtures. Currently, in the bituminous laboratory, data
are being collected on bituminous concrete mixtures conforming to
Indiana AH Type B Specifications.

                                    Respectfully submitted,

                                    Harold L. Michael, Assistant Director
                                    Joint Highway Research Project

HLM:hb

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure

ABSTRACT

Wood, Leonard Eugene. Ph.D., Purdue University, August 1956. The Stress-Deformation Characteristics of Asphaltic Mixtures Under Various Conditions of Loading. Major Professor: William H. Goetz.

A laboratory study was made of the effects of various conditions of loading upon the stress-deformation characteristics of bituminous mixtures in order to obtain a better understanding of the various basic factors that affect their performance.

The study was divided into four parts. In the first part, using the unconfined compression test on a sand-asphalt mixture, a general relationship between maximum, unconfined compressive strength, temperature, and rate of deformation was developed as follows:

$$x_0 = A^{Bx_1^{(Cx_2 + D)}} \qquad \ldots \ldots \ldots \ldots \ldots (1)$$

where x_0 = maximum, unconfined compressive stress, psi

x_1 = rate of deformation, in./min.

x_2 = temperature, $^{\circ}F$

A, B, C, D = parameters to be determined from test results.

Multiple linear regression analysis was used to evaluate the above parameters and also to evaluate the degree of association the expression represented.

The validity of this general expression for other mixtures was determined by formulating two different mixtures and evaluating the parameters of the above equation from limited results. With the established

parameters, the maximum, unconfined compressive stress was predicted for other test levels. Specimens were then tested for these levels and in most cases there was very close agreement between predicted and observed values.

The validity of the general expression for other loading conditions was determined by performing the compression tests at confining pressures of 15 and 30 psi and making similar comparisons between predicted and observed values.

In the second phase, unconfined, repeated load tests were performed. The concept was brought out that there was a stress that could be cycled a number of times without causing failure (excessive shear deformations) to occur. This stress was labeled the endurance limit. Regardless of the test level, the endurance limit appeared to have a value of approximately 25 percent of the maximum, unconfined compressive stress for that particular test level. A general relationship between temperature, rate of deformation, number of load repetitions, and applied stress was evolved as follows:

$$x_c = \left[E \cdot 10^{-\alpha(n-1)^\beta} + (1 - E) \right] x_0 \quad \ldots \ldots \ldots \ldots (2)$$

where x_c = applied, cycled unconfined compressive stress, psi

x_0 = maximum, unconfined compressive stress, psi

n = number of load repetitions necessary to cause
excessive shear deformations

E, α, β = parameters to be determined from test results.

Multiple linear regression analysis again was used to evaluate the above parameters. An expression relating the first and second phases of the study was also developed and is presented as follows:

$$\log \log x_c = A + B \log x_1 + Cx_2 + Dx_2 \log x_1 \left[E \cdot 10^{-\alpha(n-1)} + (1 - E) \right] \quad (3)$$

where x_c = applied, cycled unconfined compressive stress, psi

x_1 = rate of deformation, in./min.

x_2 = temperature, $^\circ$F

n = number of load repetitions necessary to cause failure

A, B, C, D, α, and β = constants that are dependent upon

mixture composition.

The data suggest that the elastic portion of the deformation takes place principally in the polymolecular film of asphalt which surrounds the aggregate particles.

The third phase consisted of performing confined, repeated load tests. The previous concept of an endurance limit equal to approximately 25 percent of the maximum compressive stress for any given test level was confirmed.

A series of static load tests composed the fourth phase of the study and pointed out quite vividly the difference in mixture characteristics as affected by temperature. At 140°F the mixture was quite plastic in character. At 40°F the mixture appeared to have considerable viscous resistance under static load.

It was concluded that a promising means of evaluating the adequacy of a bituminous mixture before its utilization in the field would be to perform the confined, repeated load test on a rational specimen at a temperature of 140°F and a rate of strain of 0.0005 in./in./min. The degree of confinement to be introduced cannot yet be recommended. From previous work on the strength of bituminous mixtures, it is recognized

that some degree of confinement should be used in order to distinguish those mixtures whose strengths are particularly benefited by confinement.

The reasons advanced for these recommendations are as follows: 1) at the given temperature and rate of strain the mixture would be quite plastic in character and the test should indicate the stability of the mixture during summer climatic conditions under repeated slowly moving loads, and 2) the desired property of the mixture would be to have as high an endurance limit as possible.

THE STRESS-DEFORMATION CHARACTERISTICS

OF ASPHALTIC MIXTURES

UNDER

VARIOUS CONDITIONS OF LOADING

INTRODUCTION

From the very beginning of bituminous paving construction there has been a lack of knowledge regarding the actual interrelationships existing among stability, temperature, and rate of loadings for bituminous mixtures. This lack of knowledge has led to the development of a number of so-called "empirical" tests used in the design of these mixtures. The results of these tests, when correlated with service performance, gave rise to design criteria in current use.

Field correlations, once established, could be used again and again. However, in recent years, with the advent of greater traffic intensities and higher tire pressures, the empirical tests have failed to answer adequately the question of what should constitute proper design criteria for conditions other than those that were used to establish the original design criteria.

From the wide variation of test procedures used in current design methods it is evident that there is a definite lack of understanding regarding the fundamental behavior of bituminous mixes. Most of the present day procedures leave much to be desired as to how the mixture would react under conditions of loading and temperature other than those for which the correlation was established. In some cases, design criteria are based upon test values obtained from rates of deformation and temperatures such that the mixes being tested exhibit definite elastic

properties. However, present day thinking has begun to recognize the importance of plastic action in bituminous mixtures as evidenced by the current papers of Neppe (40)[1], Mack (46), and Nijboer (47).

It is known that bituminous materials show properties which differ widely from those of elastic materials. The deformation which bituminous materials undergo is thought to be made up of two distinct parts: 1) plastic and 2) elastic. These two components may vary widely under different conditions of loading and temperature.

Field performance indicates that extreme conditions of loading for a bituminous layer occur during summer climatic conditions under a stationary load or at locations where heavy trucks with high tire pressures undergo slowing or stopping movements. The plastic component of deformation under these conditions assumes major importance. The proper mixture design should minimize this component.

While it is important that a mixture should have sufficient stability under the above conditions, it must be remembered that winter temperatures impose a situation upon the mixture such that it acts more like an elastic body. Under low temperatures, the mixture could become quite brittle.

An adequate design procedure should answer the question as to how the mixture would react under wide ranges of temperature, rates of loading, and types of loading. Based upon current design concepts, a mixture may qualify for use in a road. Under actual field conditions, it may have undesirable characteristics which should be detected before the mixture is placed.

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[1] Numbers in parentheses refer to references listed in Bibliography.

For many years, the Indiana AH type B mix has given satisfactory service both as a flexible pavement surface and as a bituminous concrete overlay. Recently, in some areas, bituminous concrete overlays made from this mix are showing signs of distress such as rutting and shoving in the wheel-tracks. It is theorized that two factors have brought about this situation: 1) Heavier wheel loads have increased the stress that the overlays experience, and 2) Increased repetitions of load have increased the accumulated permanent deformation. The combination of these two factors has brought about a situation such that plastic deformation becomes a significant factor. This plastic deformation is not recoverable; the lanes experiencing the majority of the wheel loads are permanently deformed. This action is especially noted in the vicinity of stop lights where the stresses in the overlay are applied slowly and over a longer period of time than on the open stretch of road.

To be able to devise a more rational test for determining performance characteristics of a proposed bituminous mix, it is necessary to have a relationship established between strain rate, temperature, intensity of load, and the number of load repetitions. It was for this purpose that this investigation had its inception.

REVIEW OF THE LITERATURE

Much has been written regarding the effect of temperature and rate of deformation upon various properties of compacted bituminous mixtures. Recently, several investigators have begun to recognize the importance of plastic action and repeated loads when considering the desirability of a bituminous mixture for placement in the field.

This review is divided into three sections: (a) The Influence of Temperature and Rate of Deformation on Bituminous Materials, (b) The Effect of Temperature and Rate of Deformation upon Bituminous Mixture Properties, and (c) The Effect of Repeated Loads Upon Bituminous Mixture Characteristics.

The Influence of Temperature and Rate of Deformation
on Bituminous Materials

There are many factors which have a marked effect upon the performance of compacted bituminous mixtures (1). The most important factor with respect to temperature change is the nature and content of the bituminous binder. In a dense-graded surface the bituminous material acts with the mineral filler as a plastic matrix (2). In order for the surface to adjust itself to small increments of strain induced by temperature variations, moisture changes, and traffic without cracking, the matrix must remain in a plastic condition. To resist the forces of traffic, the matrix must have adequate resistance to flow.

Asphalts take on the characteristics of true liquids at high temperatures, but at lower temperatures they exhibit properties of complex flow. Lee (2) found that a plot of deformation versus time for a constant stress on asphalt gives a curve exhibiting several interesting effects: 1) initial elastic effect, 2) structural breakdown (thixotropic region), 3) steady flow conditions, and 4) partial elastic recovery upon release of applied stress.

In a fairly recent paper Mack (3) discusses the viscous-plastic flow properties along with thixotropic and elastic effects exhibited by asphalts.

The change of consistency asphalts undergo with changes in temperature is generally evaluated by means of standard tests such as softening point, viscosity and penetration. It has been discovered that for certain temperature ranges a plot of temperature versus log viscosity results in a straight line. Schweyer, Coombs, and Traxler (4) proposed that the temperature susceptibility of asphalts be based on a percentage decrease of viscosity for 1°C rise in temperature and labeled this as the Asphalt Viscosity Index. Since the ranges of temperature where the straight line relation holds are rather limited, this index does not receive much attention.

Vokac (5) made a study of variables affecting the compressive strength of briquets of asphalt and filler. He found that the composition of the mix, the type of asphalt cement, and the temperature of the mix had a profound effect upon the temperature susceptibility of the mix. Within like mixtures, the temperature susceptibility of the mix-

ture varied directly with the temperature susceptibility of the asphaltic

cement.

Thelen (6) defined five rheological properties which he considered

basic to a proper understanding regarding the manner asphalts and simi-

lar materials react under varying temperatures and rates of shear.

These properties and their definitions are as follows:

> 1. Yield value is the intercept on the shearing stress
> axis, of the shearing stress-rate of shear curve. 2. Mobili-
> ty, the slope of this curve, is the ratio of the rate of flow
> to the net shearing stress causing it. 3. Elasticity is the
> rate of flow per unit shearing stress, when the shearing stress
> is less than the yield value. 4. Work of adhesion is the
> energy required to separate the asphalt from a superficial
> surface of unit area. 5. Vibration resistance is the num-
> ber of cycles of known acceleration, required to cause failure.

In his paper, Thelen outlines procedures whereby the above rheological

properties may be evaluated.

The effect of asphalt characteristics on the physical properties

of bituminous mixes was investigated by Lewis and Welborn (7). They

felt that the physical properties of a given mixture were dependent upon

the consistency of the asphalt in the mixture at the test temperature.

Lewis and Welborn showed that a linear relationship existed between the

log of the penetration and temperature of an asphaltic cement. They

also showed that a plot of log Hubbard Field Stability versus test tem-

perature gave a linear relationship for stabilities below 10,000 pounds.

The temperature range investigated was -25°F to 140°F.

Several investigators have attempted to predict behavior of as-

phaltic mixes from test results obtained on asphaltic cements. This is

very difficult to do under actual construction conditions. Mixing tem-

peratures, duration of mixing, and weathering all play an important role in altering the consistency of the asphaltic cement when placed in a mix.

In an investigation on the effect of mixing temperature upon penetration of recovered asphaltic cement, White (8) reported that at a mixing temperature of 225°F a 60-70 penetration asphalt cement showed a decrease of seven points in penetration after recovery from a mix while a mixing temperature of 330°F resulted in a decrease of 21 points in penetration.

Steinbaugh and Brown (57) in a study dealing with the cracked flexible pavements pointed out the importance of controlling the mixing temperature and the time of mixing in order to prevent undue hardening of the asphaltic cement. They found some types of asphaltic cements weather very rapidly.

Hubbard and Gollomb (58) reported that undue hardening of an asphaltic cement may occur at the mixing plant and also in the pavement under service conditions. They recommended that adequate controls and inspection be maintained at the plant to insure proper temperatures and proper mixing times and that the thin asphalt film present in the pavement be protected by placing a dense mixture which would reduce exposure to air.

Tucker (59) introduced the concept that asphaltic materials are not only hardened by oxidation but also by a phenomenon which he called "age hardening." This age hardening is probably due to colloidal or internal structural changes. Careful heating of an age hardened asphalt

will return it to its original consistency. Action of traffic helps retard age hardening.

Parr and Schaub (60) in a study on the effects of changes in paving asphalts upon service behavior reported that a paving asphalt having a penetration of 85 showed a loss of penetration to 50 upon being mixed at 350°F and a loss of penetration to 18 upon being mixed at 500°F. The same asphaltic cement (penetration 85) was mixed at a temperature of 340°F for varying lengths of time. After 30 seconds of mixing, the penetration had dropped to 55 and after 150 seconds of mixing, the penetration had dropped to only 50.

Lewis and Halstead (9) reported the results of a series of tests showing the effect on asphalts exposed to elevated temperatures in thin films. In some cases a large drop in penetration was noted.

Hughes and Faris (10) attempted to correlate the low temperature maximum deformability of asphalts with various standard tests such as the ductility test and the penetration test. They conclude that the penetration test and ductility test evaluate only a small portion of the deformability (i.e., the ability to bend without breaking) of asphalts.

In a later paper Hughes and Hardman (11) correlated important flow properties such as softening point, penetration and ductility with asphalt composition. The correlations were limited to products which were not highly cracked and to asphalts whose penetration was not harder than 20. Rader (12) found that there was little correlation between ductility of recovered asphalt and resistance to cracking of sheet asphalt. He also reported that "other factors being equal, it would appear that those mixtures containing the highest penetration asphalt consistent with necessary stability should prove most resistant to cracking at low temperatures."

Pauls and Welborn (13) in a study on the effect of weathering upon asphaltic materials reported that asphaltic materials vary considerably in their resistance to hardening depending upon the method of refining and the crude source, and that the thin-film oven test is a very useful tool in determining the hardening properties of asphaltic materials.

Van der Poel (38), in a study dealing with the visco-elastic properties of bitumens, reported that:

> 1. The mechanical behavior of bitumens can be described for almost the whole temperature range of practical interest by a stiffness modulus, defined as the ratio of stress to strain. 2. This stiffness modulus depends on four variables: (a) time of loading or frequency, (b) temperature, (c) hardness of the bitumen, and (d) rheological type of the bitumen. 3. Hardness of the bitumen can be completely characterized by the ring-and-ball temperature, and rheological type by the penetration index.

Van der Poel also constructed a nomograph so that the stiffness of a bitumen could be determined for any combination of temperature and rate of loading.

The Effect of Temperature and Rate of Deformation
Upon Bituminous Mixture Properties

Many different types of tests have been used over the years in order to evaluate bituminous mix properties. Skidmore (14) was an early investigator who made use of the direct shear test to evaluate the low temperature characteristics of bituminous mixtures. He used a sheet asphalt mix whose constituents were 50-60 penetration asphalt cement, filler, and sand. His test temperatures ranged from 32°F to -20°F. The rate of loading was dependent upon the specimen size. Skidmore

found that the shearing strength decreased rapidly with an increase in temperature. He concluded that more care should be exercised in selecting asphalt quantity and characteristics at low temperatures than at high temperatures.

Vokac (15) criticized the direct shear test as not being realistic. He felt that other stress combinations were present in an actual pavement.

Vokac in turn proposed that a compression type test should be used to evaluate bituminous paving mixtures (16, 17). He obtained simultaneous readings of load and deformation. From a curve of load and deformation he obtained several interesting characteristics: 1) modulus of elasticity--slope of straight-line portion of the curve, 2) elastic limit--point where straight line deviates from curve, 3) ultimate stress--maximum stress specimen could withstand, 4) modulus of permanent deformation--slope of load-deformation curve past the ultimate stress. Vokac found that there appeared to be a relationship existing between temperature and ultimate stress and rate of loading and ultimate stress and developed equations relating the paired variables but did not relate the three variables into one equation. Vokac also investigated a method of using impact testing of bituminous mix specimens (21). He used a Page impact machine. The test temperatures ranged between 32°F and 140°F. He felt that this type of test would be very valuable in evaluating brittleness and toughness of bituminous mixtures.

Vokac broadened his study to include a correlation of physical testing with service behavior of bituminous mixtures (18, 19, 20). Specimens were made from a large number of mixtures and subjected to

the Skidmore Shear Test (14), the Hubbard-Field Stability Test (50),
the Impact Test (21), and the Compression Test (16). He found that of
all the above tests, the Compression Test was the most sensitive in
predicting the manner in which a mix would withstand actual field con-
ditions. The compressive stress and the elastic limit were found to be
the most important characteristics in bituminous mixtures. These char-
acteristics are closely related and indicate the magnitude of stress a
surface mixture can withstand without serious deformation.

A criticism of Vokac's work has been made by Pfeiffer (22). He
felt that Vokac's test specimen had a height-diameter relationship such
that a rational test result was not obtained. Vokac's specimens were
generally two inches in diameter and one inch in height. To be a
rational specimen, the height should have been four inches in order to
minimize end friction.

Rader (12, 23, 24) was another early investigator in the field of
physical properties of bituminous mixtures and how they are affected by
temperature changes. He noted that asphaltic mixtures are usually
plastic at normal temperatures. At lower temperatures he observed that
the mixes became more brittle. Rader hypothesized that the pavement
cracking at low temperatures was due to tensile stresses set up by con-
traction. To evaluate the characteristics he felt were important,
Rader utilized the flexural test on beams and obtained a modulus of
rupture and a modulus of elasticity in flexure.

The rate of loading in Rader's tests was 0.05 in./min. and the
temperatures ranged from -77°F to 0°F. He noted that asphaltic mixes
became stiffer as the temperature was lowered. Rader correlated his

laboratory study with field observations. He concluded that for good low temperature performance a mix should have a high modulus of rupture and a low modulus of elasticity.

Observations similar to those reported by Rader were reported by Raschig and Doyle (25, 26). They conducted flexural tests on beams loaded as a cantilever. The test temperatures varied from -5°F to 38°F. Their results showed that as the temperature was decreased, the modulus of elasticity and the modulus of rupture both increased. Raschig and Doyle performed tests on asphalt samples extracted from pavements and reported the penetration and ductility values of the extracted asphalt had the best correlation with the modulus of elasticity and the modulus of rupture values.

Hillman (27) reported the results of bending tests patterned after Rader (23). His tests were made on beams composed of a sheet asphalt mix. He varied both the rate of loading and the temperature. By taking simultaneous load-deflection readings, it was possible to construct load-deflection curves. Since there was no straight line portion in any of the curves, Hillman calculated a secant modulus of elasticity. It was as follows:

$$E = \frac{\ell^3}{4bd^3R}$$

where E = modulus of elasticity, psi

R = avg. rate of deflection, inches per pound of load

ℓ = length of simply supported span, inches

b = breadth of sample, inches

d = depth of sample, inches

His formula for the modulus of rupture was as follows:

$$S = \frac{3P\ell}{2bd^2}$$

where S = modulus of rupture, psi

P = breaking load, pounds

b, d, and ℓ are the same as in the formula for the

modulus of elasticity.

Hillman reported that an increase in the rate of loading, for a fixed temperature, resulted in an increase in both the modulus of elasticity and the modulus of rupture, and that as the temperature was lowered an increase in the modulus of elasticity and the modulus of rupture was observed. The results of beams taken from pavements indicated that the highest modulus of rupture came from pavements which were considered to be in the best condition.

Lonsdale and Wilson (28) made a study where they were able to evaluate the shear modulus of a specimen molded in the shape of a rod. The test temperatures were varied from -12°C to 25°C. They observed that as the test temperature was increased, a decrease in the shear modulus was obtained. Lonsdale and Wilson noted that increasing the filler content increased the shear modulus.

Hughes and Farris (10) reported that

...work carried out at our laboratories on the low temperature elasticity and viscosity of asphalts indicated that both of these closely inter-related properties contributed in a large measure to the overall mechanical effects observed. This complex variation of viscosity and elasticity with temperature and with rate of deformation has been studied by many investigators. The difficulty of studying separately the inter-related fundamental physical properties of the asphalt system led to the study of the maximum deformability (i.e., the ability to bend without breaking) which suggested itself as a possible critical criterion.

The tests were conducted on beam specimens at two rates of deformation: three inches per second and 0.07 inches per hour, and two temperatures: 0°F and 32°F. At the high rate of deformation, temperature did not affect the deformability of the mix to the extent that would have been predicted by the effect of temperature on related physical properties such as the softening point and the change in penetration values. The authors concluded, "that the high deformation rate forces were not the critical ones in pavement performance, causing us to direct our attention to the low deformation rate forces."

The low rate of deformation test series indicated that at 0°F deformability was less than half the corresponding values at the high rate. The deformability for the low rate was greater at 32°F than at 0°F. In correlating the laboratory results with field performance, it was found that mixes exhibiting low deformability in laboratory tests had poor service records while mixes which had a large deformability had good service records.

Hughes and Farris conclude, "low deformation rate effects are much more critical than the high deformation rate traffic-type loading even at these low temperatures where one would expect asphalts to be more brittle."

It should be possible to use a triaxial test to evaluate the variation of mix properties under different temperatures and rates of deformation. The background of triaxial testing of bituminous mixtures has been thoroughly developed by many investigators (29, 30, 31, 32).

Endersby (29) states that the triaxial test is suitable for studying a plastic material such as a bituminous mixture where the resistance

to deformation is made up of two components: 1) friction and inter-
locking of the aggregate, and 2) resistance of the binder to shearing.

Nijboer (30, 33) used the triaxial apparatus in a rather lengthy
study of bituminous mixtures. He reported that temperature had quite
an effect on the initial resistance or cohesion of the mix. Lowering
the temperature increased the initial resistance. Nijboer also reported
that rate of deformation had minor effects upon the initial resistance.
The angle of internal friction was observed to be a maximum at the
higher temperatures. Nijboer obtained on a sheet asphalt an angle of
internal friction at 50°F of 26°. At 104°F the angle of internal fric-
tion had increased to 31°.

The Bureau of Public Roads (34) has evolved a new type of triaxial
cell that would lend itself nicely to a study of temperature effects
upon bituminous mixture characteristics.

There are two types of triaxial testing in general use: the "open"
system and the "closed" system. Smith (31) reported concerning the
"closed" system that since readings are taken when the rate of strain
is essentially zero the effects of temperature are eliminated. The zero
rate of strain would be valid for design only if static loads are con-
sidered to be critical; however it is known that the viscous character-
istics of bituminous materials play a great part in resisting heavy,
moving, wheel loads and temperature has a marked effect upon this re-
sistance to displacement.

The work of Lee and Markwick (35) is of interest by virtue of the
interesting test methods utilized. They used a variation of the tension
test. A constant stress was applied to the specimens and the resulting

deformation was measured at various time intervals. Another variation
was also used whereby the rate of strain was held constant by increasing
the load. The maximum load was recorded. The constant rate of strain
tests proved very useful in determining "ageing effects" of the samples.

The authors found that a plot of log strain rate versus log stress
resulted in a straight line plot for a temperature of 0°C.

Pfeiffer (22) used a tension test to evaluate bituminous mix prop-
erties under varying temperatures and rates of deformation. He found
that tensile strength increased as the rate of deformation increased
and that a decrease in temperature caused an increase in tensile strength.
Pfeiffer felt that tension and compression tests of bituminous mixtures
were satisfactory for investigating the effect of different mixture com-
positions but did not permit one to draw accurate conclusions as to the
manner in which the mix would react in the field.

Eriksson (36) investigated the effect of different temperatures
and rates of elongation upon the tensile strength of bituminous mix-
tures. The elongation rates varied from 0.0001 cm./sec. to 5 cm./sec.
and the temperature varied from -70°C to 60°C. In evaluating the effect
of temperature there appeared to be a point of discontinuity in the
curve. This occurred at a temperature of approximately 0°C and gave a
maximum tensile strength at that temperature of 50 kg./cm.^{2}. The ten-
sile strength increased as the temperature was lowered until 0°C. At
temperatures below 0°C the tensile strength decreased from its peak
value as the temperature was lowered.

It was reported that increasing the rate of elongation resulted
in an increased tensile strength. Eriksson noted two types of rupture—

plastic rupture and brittle rupture. At low temperatures and high rates of elongation, brittle rupture was observed while at high temperature and low rates of elongation plastic rupture was noted.

Itahura and Sugawara (37) made an investigation of the stability of bituminous mixtures at various temperatures using a Page impact testing machine to evaluate mixture toughness. The toughness for any one asphalt content was found to decrease as the temperature was lowered.

Mack (46) investigated the deformation mechanism and bearing strength of bituminous pavements. He defined the bearing strength as "the maximum load per unit area which a bituminous pavement can carry without causing initial failure." Mack recognized three types of deformation and labeled them as instantaneous, retarded elastic, and plastic. Of the three, plastic deformation was of the most importance. He used a static load and recorded the deformation at different time intervals. Mack reported that the bearing strength appeared to be independent of confinement but was affected by asphalt content and temperature. The bearing strength decreased as the temperature was raised.

Nijboer (39) reported the results of a series of tests dealing with the effect of temperature and rate of loading upon the tensile strength of bituminous mixes. He showed that a bituminous mixture becomes stiffer as the temperature is lowered and the rate of loading is increased. Nijboer brings out the concept of plastic deformation which occurs under long duration loading. He divided the mixture's resistance to shear into three parts: 1) frictional resistance, 2) initial resistance, and 3) viscous resistance.

Heppe (40) made an investigation into the mechanical properties of bitumen-aggregate mixes and how they are affected by binder characteristics. He concluded,

> ...that the best service can always be expected in practice where bitumens of high softening point number are used—that service performance at low and high temperatures is far more dependent on the inherent characteristics of the binder than at normal temperatures—that mixtures prepared with low softening point number bitumens are far more sensitive to a change in the proportion of the binder than equivalent high-index bitumens.

Waller (41) reports the effect of varying temperatures and rates of deformation upon the stability of bituminous mixes tested in accordance with A.S.T.M. Method of Test for Compressive Strength of Bituminous Mixtures (D1074-52T). In order that the stability value should not vary more than five percent, the temperatures must be controlled at 77°F ± 1.2°F and the rate of deformation should be controlled at 0.2 in./min. ± 0.05 in./min.

The Effect of Repeated Loads Upon
Bituminous Mixture Characteristics

Nijboer (39) stated "it should be taken into account that deformation or accumulated deformation on repeated loading may be the criterion to be used." He reported the results of fatigue tests on beams molded from bituminous mixtures and concluded that as the bending strength is decreased the number of repetitions the beam will withstand without failing is increased. There appeared to be a minimum bending stress below which a beam would withstand a large number of load repetitions without failure. Nijboer performed his tests at 10°C because "deforma-

tion of a road carpet under a given load is greatest during the spring

break-up. The temperature of the carpet may then be about $10^{\circ}C$."

Mack (44) in a comprehensive study on the rheology of bituminous

mixtures, reported on a study of the effect of repeated loads that

successive loadings caused work hardening and resulted in an increase

of time necessary for any one load application to bring about a certain

deformation. This means the mixture became more resistant to flow.

It was also shown that as the temperature of test was increased, less

time was necessary for a given load to bring about a certain deforma-

tion. Mack states,

> ...relationship between the flow properties of the bituminous
> mixtures and those of the asphalts used is also reflected in that
> the coefficient of work-hardening and degree of plasticity of
> bituminous mixtures increases at a given deformation and temper-
> ature with the degree of plasticity of the asphalts used.

Mack, in a later study (45), investigated the effect of cycling

one large load and the effect of starting to cycle some small load and

with each cycle to increase the load some fixed amount until the magni-

tude of the large load used in the first phase of the study was obtained.

He concluded that larger deformations can be accumulated by cycling one

fixed large load than by increasing the load each cycle. This is due

to a time function which "indicates that under a large load a greater

strain is obtained at the end of the plastic deformation than when first

a smaller load is applied for some time and then the load increased to

the value of that of the larger load."

Mack divided the deformation of bituminous mixtures under load into

a nonrecoverable plastic deformation and recoverable deformation. He

showed that the magnitude of these deformations was a function of time.

There is very little additional information on repeated loads in the literature on bituminous mixtures. It was felt that some of the work in Soil Mechanics dealing with repeated loads could be reviewed. Tschebotarioff (42) reported the results of a study on the effect of repeated loads on soil characteristics. He found that the frequency of loading had little effect on plunger penetration. Void ratio was an important factor. As the void ratio of the sample increased, the plunger penetration for a given load and number of load repetitions also increased. Tschebotarioff noted that there were some loads that could be cycled a great number of times with very little change in plunger penetration. At larger loads the plunger penetration continued to increase.

Seed, Chan, and Monismith (43) reported the results of a repeated load study on clay. They found that there appeared to be a relationship between the applied stress and the number of load applications necessary to reach some failure criterion. Several plots were developed relating axial strain versus load applications at various stresses.

It is apparent that the more recent investigations are concerned with the plastic action and rheological characteristics of bituminous mixtures. The next step would appear to be the utilization of repeated load information in a logical mixture design procedure that would insure its adequacy when placed in the field.

MATERIALS

The mixture chosen for the major portion of this study was a sand-asphalt mixture. This permitted the use of relatively easy molding procedures and made it possible to utilize the testing equipment available in the laboratory immediately. In order to check portions of the findings based upon the sand-asphalt mixture for wider application in the field of bituminous mixtures, specimens were molded from a crushed limestone having a 1/2 inch maximum size aggregate.

A complete description of the aggregates and asphaltic cement used in this study are included in two separate sections.

Aggregates

The sand used in this study was a local, natural material obtained from a river terrace. The gradation chosen met the requirements of A.S.T.M. D978-54 Standard Specifications for Asphaltic Mixtures, for Sheet Asphalt Pavements, Surface Course, Grading No. 2 (47) and Asphalt Institute 100-XI Sheet Asphalt Surface Course (48). The sieve analysis of the gradation is presented in Table 1 and depicted graphically in Figure 1.

As the terrace sand was deficient in the minus 200 material, this fraction was obtained by adding pulverized limestone. By altering the character of the minus 200 material, two mixtures were formulated. These two mixtures are designated as I and II. Mixture I was obtained

Table 1

Sieve Analysis of Sand-Asphalt and

Indiana AH Type B Surface Course Mixtures

(Percent by Weight)

Sieve			Grading	
Passing	Retained		Sand-Asphalt	AH Type B
---	1/2 inch		0	0
1/2 inch	3/8 inch		0	9
3/8 inch	No. 4		0	29
No. 4	No. 8		0	20
No. 8	No. 16		7	12
No. 16	No. 50		34	17
No. 50	No. 100		27	6
No. 100	No. 200		15	3
No. 200	---		17	4

GRADATION CURVE

FOR

SHEET ASPHALT MIXTURE
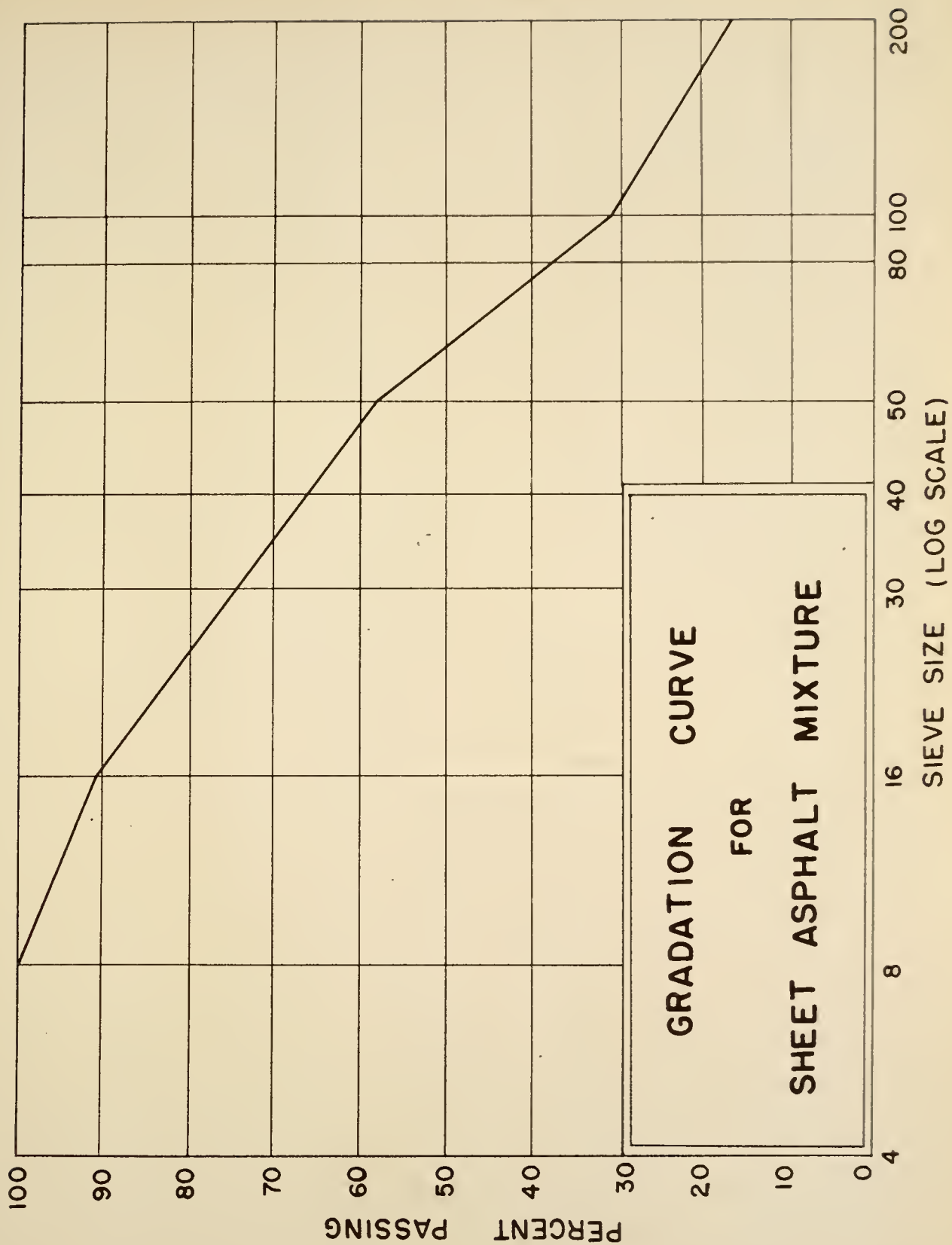
FIGURE I

by utilizing a commercial limestone mineral filler. Mixture II was obtained by using the material passing a No. 200 sieve as obtained by sieving an agricultural limestone.

The bituminous concrete mixture containing the 1/2 inch maximum size aggregate used crushed limestone obtained from a quarry at Greencastle, Indiana as both the coarse and fine aggregate. The gradation chosen for this mixture, which is designated as Mixture III, meets the Indiana AH type B Surface Course requirements (49). The sieve analysis of this gradation is presented in Table 1 and depicted graphically in Figure 2.

Specific gravity and absorption tests conforming to A.S.T.M. designations C127-42 and C128-42 were conducted on the aggregates. The test results are presented in Table 2.

The control of the gradations was obtained by drying the aggregate and recombining it by weight in the desired proportions.

Asphaltic Cement

Only one asphaltic cement was used throughout this study; it was a No. 65 Paving Cement (A.S.T.M. Penetration Grade 60-70) obtained from the Texas Company, Port Neches, Texas. Several standard A.S.T.M. tests were conducted on the asphaltic cement in the laboratory. The results of these tests are shown in Table 3.

The asphalt content used in the sand-asphalt mixture was obtained by using the Hubbard-Field design procedure (50). The data obtained by this method are presented in Appendix A. The asphalt content for the Indiana AH type B Surface Course gradation was selected from field

GRADATION CURVE FOR
INDIANA AH TYPE (B) SURFACE MIXTURE
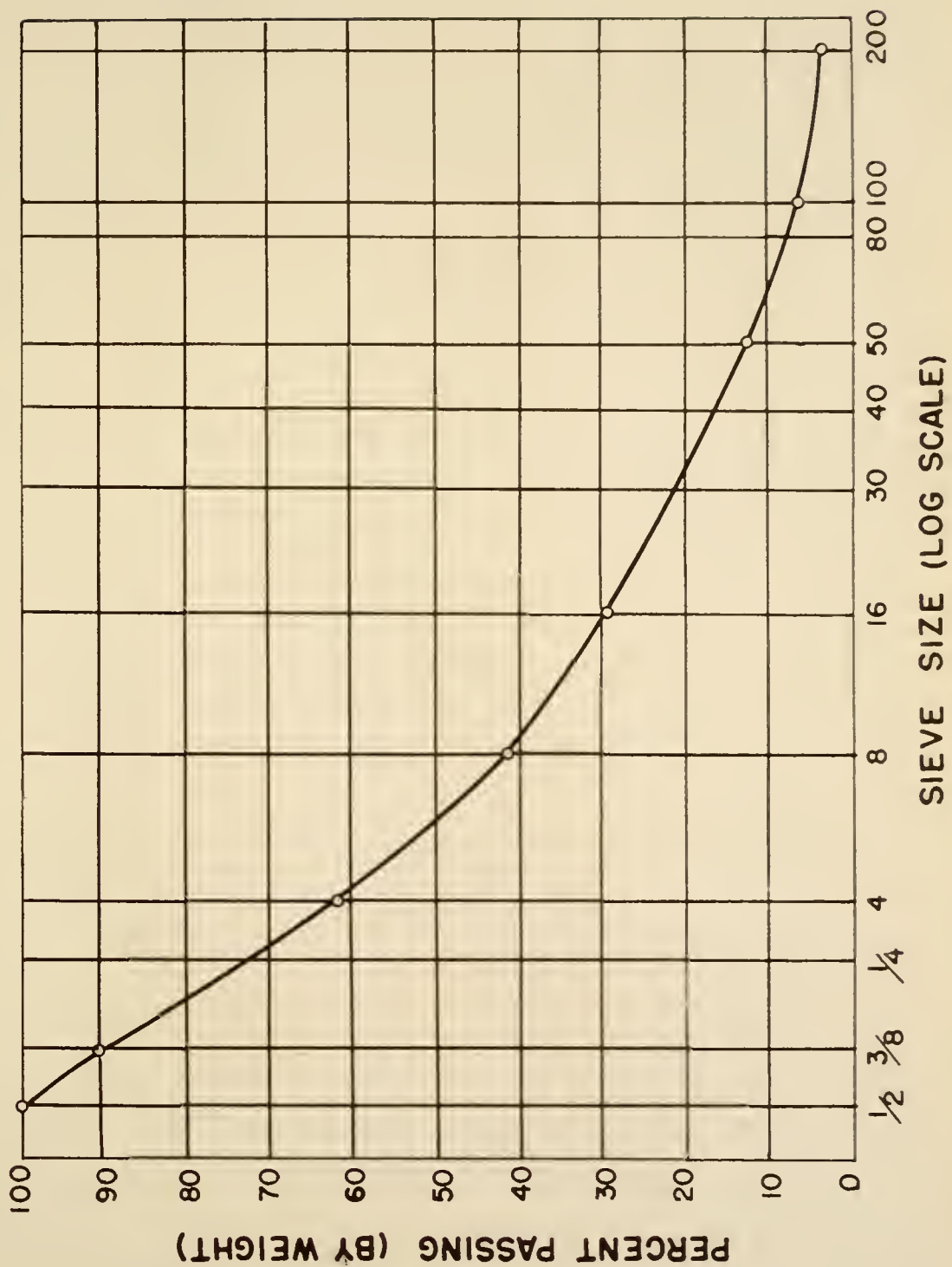
FIGURE 2

Table 2

Physical Properties of Aggregates

Aggregate	Bulk Sp. G.	Apparent Sp. G.	% Absorption
Greencastle Limestone	2.63	2.71	1.39
Lafayette Sand	2.54	2.67	2.04
Mineral Filler	—	2.73	—

Table 3

Physical Properties of Asphaltic Cement

Test	Value
Penetration (100 gms, 5 sec., 77°F) 1/100 cm.	66
Specific Gravity 77°F	1.015
Ductility (77°F, 5 cm/sec.) cm.	150 +
Solubility in $C Cl_4$, %	99.8 +
Softening Point, °F	125

experience.   The asphalt contents of the two gradations are shown in
Table 4.

Table 4

Asphalt Content of the Two Mixtures

	Asphalt Content Based upon Weight of Aggregate
Sand-Asphalt Mixture	9.0
Indiana AH type B	6.95

OUTLINE OF INVESTIGATION

A fundamental part of this investigation was the selection of a test method for evaluating strength and deformation characteristics of a bituminous-aggregate mixture while undergoing repeated loads. In the area of testing bituminous-aggregate mixtures there appeared to be a lack of knowledge regarding the fundamental relationships involved among the following variables: temperature, rate of deformation, and stability. It was decided to first establish this relationship.

For the case of repeated loads a fourth variable was added, namely, the number of load repetitions necessary to reach some failure criterion under a set of fixed test conditions. To make the investigation more realistic from the standpoint of actual field performance in a bituminous pavement where some degree of confinement is present, several levels of lateral support were included.

It was felt that a better insight as to the manner in which bituminous-aggregate mixtures react under load could be gained by including a static load test in the study.

Purpose and Scope

To establish a relationship among the temperature, the rate of deformation, and the stability of a bituminous-aggregate mixture, a series of specimens must be tested at varying rates of deformation and temperatures. Stability values at these levels must be ascertained.

By using regression analysis, a mathematical model can be determined that best expresses the desired relationship. The partial correlation coefficients of the analysis can give the amount of association among the several variables.

In the present day design methods there is a wide variation in the rate of deformation and temperature used in the various test procedures. It is felt that a general relationship of stability, rate of deformation, and temperature could be applied to a wide range of test methods. To establish this relationship, only the parameters would have to be evaluated and this could be done with a limited number of tests. This relationship, once established, would yield information as to the manner in which the mix would react under widely varying conditions of rate of deformation and temperature.

With respect to studying the effect of repeated loads, it was felt that there would be some combination of applied stress and number of load repetitions at any one test level that would lead to a failure as defined by some suitable criterion.. This failure criterion would be defined as the number of loading cycles beyond which excessive shear deformations are observed. By obtaining deformations at each loading cycle and plotting these data, one should be able to locate the point of excessive shear deformation and determine the number of loading cycles necessary to cause this condition.

From this information, and expression relating temperature, rate of deformation, applied stress, and number of load repetitions could be derived. This relationship would be quite valuable if under lateral confinement it still held true. It would mean that a mixture could be

evaluated in the laboratory in an unconfined-repeated load test and a prediction made regarding the manner in which the mixture would behave in a pavement.

It is known that a severe condition of loading for a flexible pavement or bituminous mixture is a stationary load. In order to make a complete study of different loading conditions of a compacted bituminous mixture, a test should be devised that would attempt to duplicate the effect of a stationary load.

Testing Methods

Many test methods have been proposed by which various properties of bituminous paving mixtures can be evaluated: triaxial test (1), Hubbard-Field test (50), Hveem Stabilometer test (51), and the Marshall test (52). For the purpose of this study it was felt that a relatively simple test which enables one to obtain stress-deformation data at various rates of deformation and temperatures was desired. This led to the choice of the unconfined compression test using a rational-sized specimen. The unconfined compression test was one of the earliest tests proposed for evaluating mixture properties (16, 17). More recently, the triaxial test has received rather wide acceptance since it enables one to determine two important mixture characteristics: the angle of internal friction and the cohesion. The unconfined compression test may be thought of as a special case of the triaxial test where the confining pressure is zero. Figure 3 shows a typical stress-deformation plot and notes the items of interest for this study.

RELATIONSHIP OF STRESS AND DEFORMATION
FOR AN UNCONFINED COMPRESSION TEST
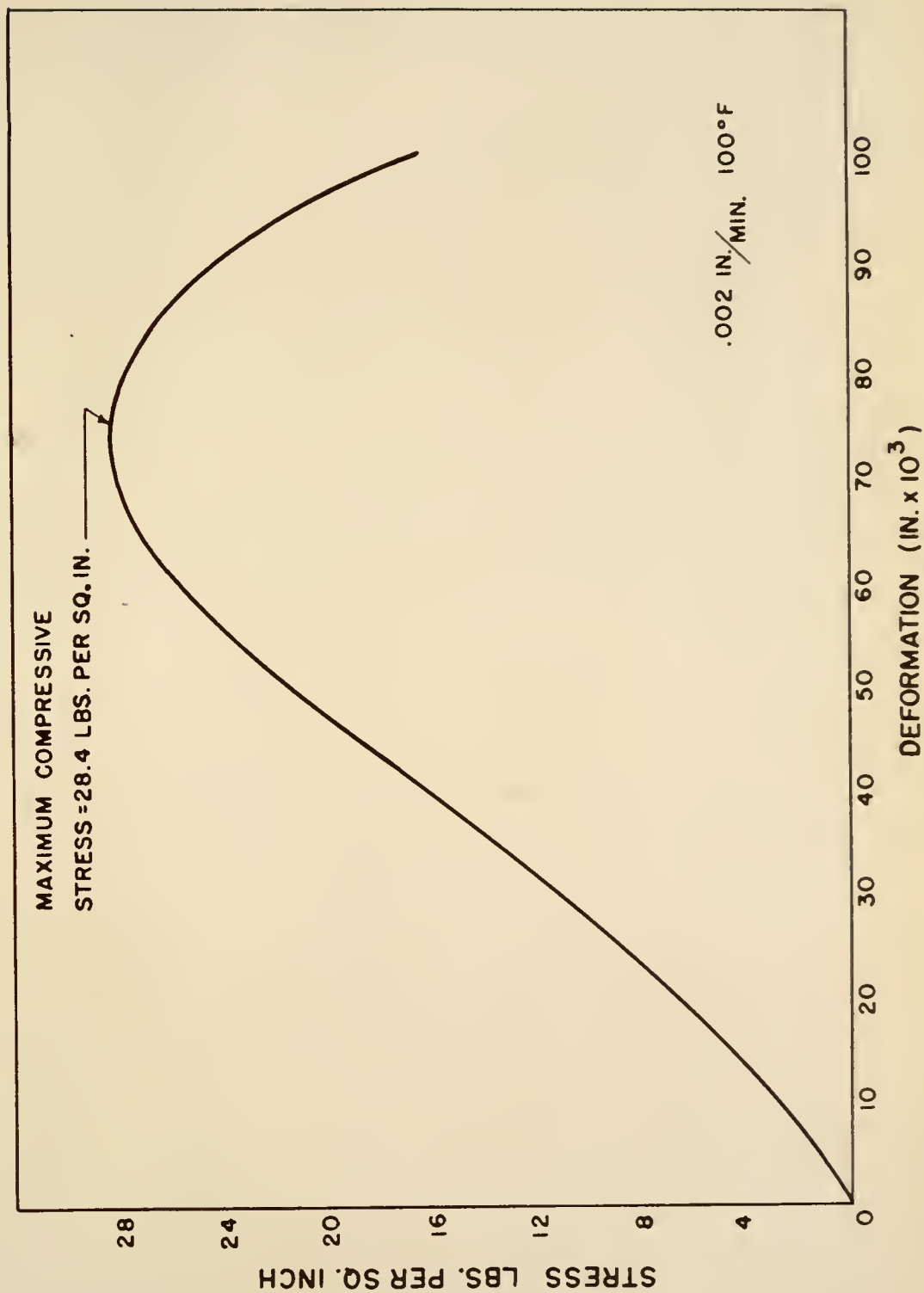
FIGURE 3

An unconfined compression type test would also lend itself to the study of repeated loads and their effect upon mixture properties. This would enable one to maintain a continuity of concept through the first two phases of the study.

For studying the effect of lateral confinement which is known to exist in actual road structures, it was decided to use the triaxial test and determine the maximum compressive stress under varying degrees of confinement and at different test conditions. To investigate the effect of repeated loads in the confined state, the triaxial cell was used to introduce lateral pressures.

The results of a stationary load upon a compacted bituminous mixture could be determined, to a great extent, by using a static loading test of the type utilized by Mack (45) in several of his studies.

APPARATUS AND PROCEDURE

The procedures used in this study will be discussed in connection with the rather general processes:

(1) Preparation of aggregate

(2) Preparation of test specimens

(3) Unconfined compression test

(4) Unconfined, repeated load test

(5) Confined, repeated load test

(6) Static load test

The apparatus used in each process is discussed under the separate headings.

Preparation of Aggregate

The following apparatus were used in preparation of the aggregate: "Gilson" mechanical sieving machine, Tyler Company's "Ro-Tap" Testing Sieve Shaker, U.S. Standard sieves, sizes 1/2 inch, 3/8 inch, Nos. 4, 8, 16, 50, 100, and 200, and a torsion balance manufactured by the Torsion Balance Company with a capacity of 4.5 kg.

The mineral aggregate was obtained directly from the source, brought to the laboratory, air dried, and sieved into the respective sieve fractions. These steps were necessary in order to insure similarity of grading among the various specimens.

The Gilson mechanical sieving machine was used to separate the coarse aggregate into the required aggregate fractions of 1/2 inch to

3/8 inch and 3/8 inch to No. 4. A Tyler Ro-Tap Testing Sieve Shaker
and U.S. Standard sieves, sizes No. 4, 8, 16, 50, 100 and 200, were
used to separate the fine aggregate into the required fractions. The
separate fractions of aggregate were stored in covered containers until
needed for batching.

In preparation for molding specimens, the separate aggregate frac-
tions were recombined by weight in the correct proportions as given in
Table 1. It was desired to form all specimens at a fixed density. The
size of the sand-asphalt mixture specimens was set at two inches in
diameter by four inches high. The bituminous concrete specimens were
set at three inches in diameter and six inches high. In order to main-
tain uniform densities, the same amounts of material were added to the
mold. For the sand-asphalt mixture, 462 grams of mixture were used each
time. For the Indiana AH type B Surface Course, 1590 grams of mixture
were used for each specimen.

Preparation of Test Specimens

For preparing the sand-asphalt mixture specimens, the following
apparatus were used: Peerless gas oven with oven heat control, Freas
electric oven with heat control, assembled compaction mold, torsion
balance, porcelain bowl, metal spoon, 1/2 inch round steel rod, hy-
draulic compaction device, and an Ames Dial device for controlling
specimen height.

For preparing the Indiana AH type B mixture specimens, the appara-
tus used were the same as those listed above with one change; the mix-
ture was blended with a modified Hobart mixer with a flat bottom mixing
bowl and blade rather than by hand mixing in a porcelain bowl and metal
spoon.

Before the actual molding was started, the asphalt and the molding equipment were prepared for use. The asphalt was received from the refinery in five gallon cans. The required amount of asphalt needed for molding two specimens was taken from the five gallon can by cutting with a heated, metal spoon and placed in a single quart sauce pan. About 30 additional grams of asphalt were placed in the pan to allow for the asphalt which adhered to sides of the pan.

The steel compaction molds as shown in Figures 4 and 5 were cleaned and the sides that were to be in contact with the heated mixture were covered with a light film of oil. The mold was then assembled.

The detailed procedure for preparing sand-asphalt mixture specimens, after the aggregate, asphalt, and equipment were prepared, was as follows:

1. The pans of aggregate, each containing the aggregate needed to form one specimen, were placed in the Freas electric oven whose temperature was set at 335°F about four hours before molding was to start. At the same time, the assembled compaction mold, the two pistons, the porcelain bowl and the metal spoon were placed on the second shelf of the Peerless gas oven whose regulator was set at 400°F.

2. Prior to molding, the pan containing the asphalt and an armored thermometer was placed on the top shelf of the gas oven and heated until the temperature of the asphalt was $285° \pm 10°F$. Care was taken not to overheat the asphalt.

3. The porcelain bowl, whose weight was previously tared, was placed on the torsion balance. The heated aggregate was added to the bowl and its weight checked.

Figure 4.   Compaction Mold for Sand-Asphalt
Mixture with a Typical Specimen
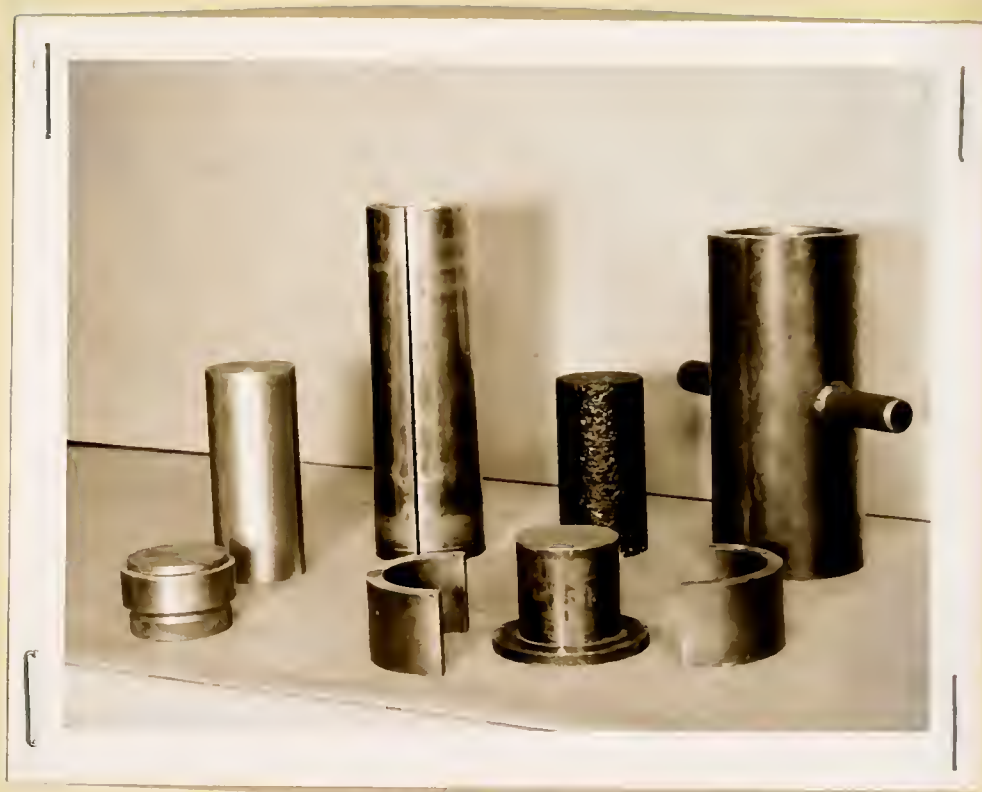
Figure 5.   Compaction Mold for Indiana AH Type B
            Surface Course Mixture with a Typical
            Specimen

4. The weight representing the amount of asphalt to be added was placed on the balance. Hot asphalt was added until the weight was balanced.

5. The mixture was stirred by hand with a metal spoon in the heated porcelain bowl for a period of two minutes.

6. After mixing, the compaction mold and the lower piston were removed from the oven and placed on the table. The compaction mold was supported on split rings. This permitted single plunger action at the beginning of compaction.

7. The mold was filled in three equal lifts. Each lift was rodded 30 times with a 1/2 inch round steel rod that weighed 1.4 pounds. After rodding the last lift, the upper piston was placed in the mold.

8. The filled compaction mold was centered in the hydraulic compaction device. For a complete description of the hydraulic compaction device see Reference 53. The tonnage selector was set at 15 tons. With the split rings still in place, a seating load was applied for a short period of time and released. The split rings were removed in order to permit double plunger compaction. Pressure was applied by means of the hydraulic jack to both ends of the mold until a specimen height of four inches was obtained as shown by the Ames Dial indicator (See Figure 6). The applied load was a variable, dependent upon the pressure necessary to obtain the above-mentioned four inches. The final applied load was maintained for two minutes and then released.

9. The mold containing the compacted specimen was removed from the hydraulic compaction device and the bolts removed from the flanges. The lower piston was removed. The upper piston was tapped several times
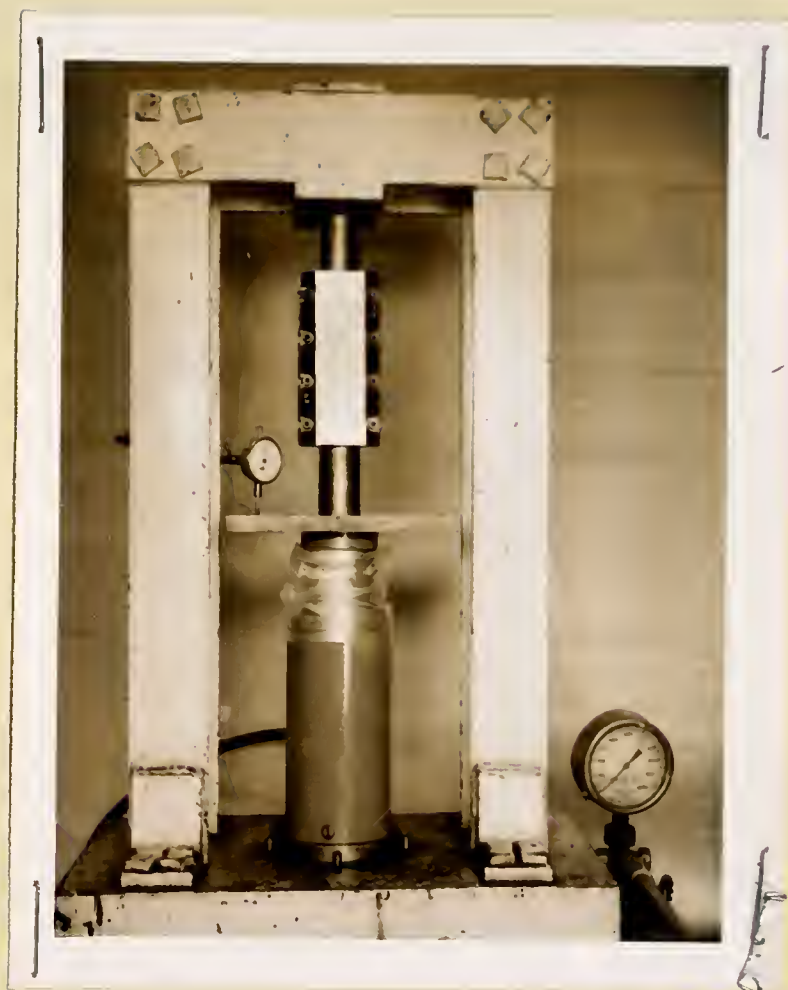
Figure 6.   Compaction Equipment Showing Ames
Dial Device

with a leather-headed mallet to break the adhesion between the specimens and the sides of the mold and to force the specimen to move a short distance. The specimen was then removed from the mold and placed on a flat plate. Typical specimens are shown in Figures 4 and 5.

10. Before testing, each specimen was weighed on a torsion balance that was accurate to the nearest 0.1 gram. The height was determined to the nearest 0.01 cm. These values were used for computing specimen densities as a check for uniformity. All specimens were then stored in a freezer at a temperature of 10°F until approximately four hours before testing.

Forming specimens from the Indiana AH type B Surface Course mixture (maximum aggregate size 1/2 inch) required several changes in the preceding preparation of specimens. Due to the aggregate size, the specimen dimensions were changed to three inches in diameter by six inches high. The compaction mold for this size specimen is shown in Figure 5. The main feature of the mold is the tapered, split sleeve. The larger specimen size required more material than could be adequately mixed by hand. For blending the materials, a modified Hobart mixer, having a side scraper, a flat mixing blade, and a flat bottom mixing bowl was used. (For a complete description see Reference 53.)

Unconfined Compression Test

The unconfined compression test was used to obtain results that would lead to establishment of a relationship among the following variables: maximum compressive stress, temperature, and rate of deformation.

The apparatus used in this test are as follows: Riehle testing machine (50,000 pound capacity) modified with a Graham variable speed drive, 500 watt immersion heater, Voltbox voltage regulator, Power-Stir electric stirrer, and a water bath.

The test set-up is shown in Figure 7. The compression load was measured by a proving ring which was attached to the movable head of the testing machine. The rate of descent of this head was controlled by the Graham variable speed drive. A wide range of constant speeds (from 4.0 inches/minute to 0.001 inches/minute) was possible.

Control of the test temperatures of 100°F and 140°F was possible by using the immersion heater and the voltage regulator. The test temperature of 40°F was easily maintained by the addition of ice to the water bath.

Before testing, the specimens were maintained at the test temperature for 30 minutes. The detailed procedure for testing was as follows:

1. The test specimen in the temperature controlled water bath was centered in the testing machine and the head brought down in contact with the specimen.

2. An Ames dial was placed such that measurements could be taken from the top of the specimen. This permitted the operator to not only control the rate of deformation but also measure the total deformation which had occurred at any one load increment.

3. The variable speed drive was set to the desired rate of deformation. As the test progressed, simultaneous readings of the load and
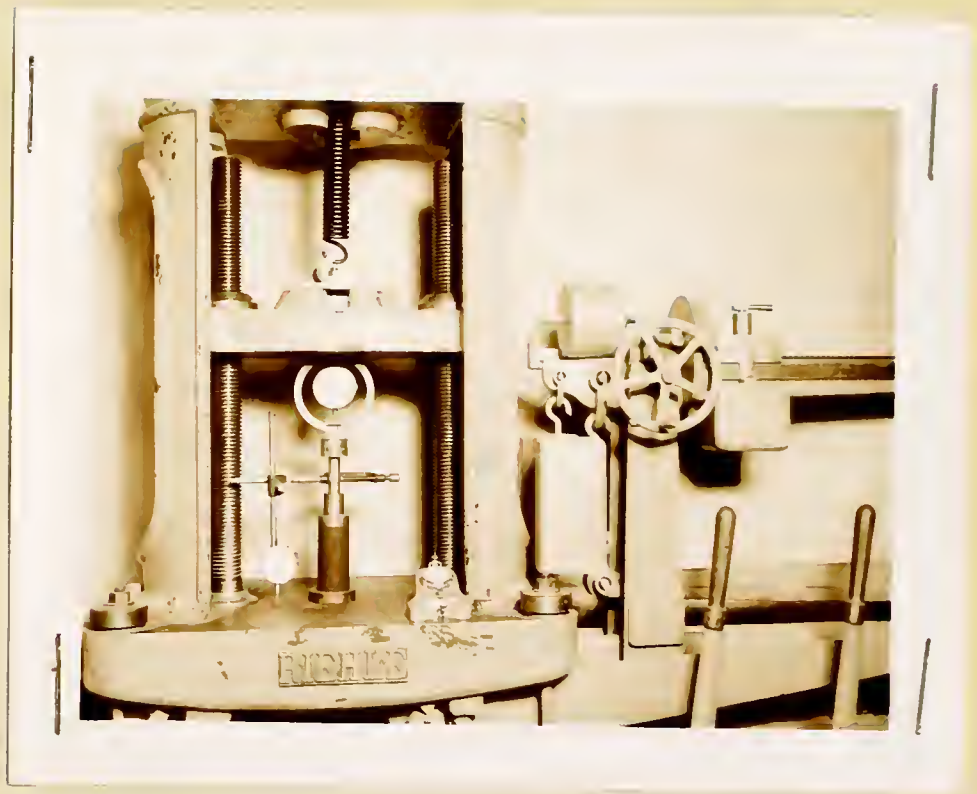
Figure 7.   General View of Unconfined Compression
             Test Apparatus Minus Water Bath

deformation dials were recorded. A typical data sheet is shown in Figure 8. Loading was continued until the maximum compressive load was reached.

4. At the end of the test the direction of the loading head was reversed and the load removed.

Unconfined Repeated Load Test

The unconfined, repeated load test was run to obtain results necessary to establish a relationship among the following variables: applied stress, temperature, rate of deformation, and number of load applications necessary to reach some failure criterion.

By means of preliminary investigations it was determined that a plot of deformation versus log load repetitions gave a straight line at the beginning of the test but with an increased number of load repetitions at certain stress levels the plot deviated from the straight line. A sharp increase in deformation for a given load cycle was noted. This deviation from the straight line plot was designated as the failure criterion.

The apparatus used in this test are as follows: Riehle testing machine (50,000 pound capacity) modified with a Graham variable speed drive, 500 watt immersion heater, Voltbox voltage regulator, Power-Stir electric stirrer, Black Hawk hydraulic jack (50 ton capacity), and a water bath.

The repeated load sequence was performed by utilizing a combination mechanical and hydraulic system. The test set-up is shown in Figure 9. The rate of deformation was controlled by the mechanical

DATA SHEET

Triaxial Test - Bituminous-Aggregate Mixtures

Specimen No. _____ I - 7 _____   Rate of Deformation _0.002 in/min._

Date Tested _10-7-55_____   Temperature _____100°F_____

By _____L.W. & R.P._____   Lateral Pressure _____0 psi___

Proving Ring _____500 lb._____   Area _____3.14 in²_____

Strain Dial 0.001	Load Dial Read.	Load, Lbs.	Strain In./in.	Correc. Area	Stress, Lbs./sq. in.
0	0				
5	8				
10	16				
15	26				
20	38				
25	49				
30	61				
35	73				
40	88				
45	103				
50	119				
55	131				
60	142				
65	153	86	.0163	3.1937	26.43
70	159	90	.0175	3.1976	28.15
75	161	91	.0188	3.2018	28.42
80	159	90	.0200	3.2057	28.07
85	150	85	.0213	3.2100	26.48
90	136	76	.0225	3.2139	23.65
95	119				
100	100				

FIGURE 8
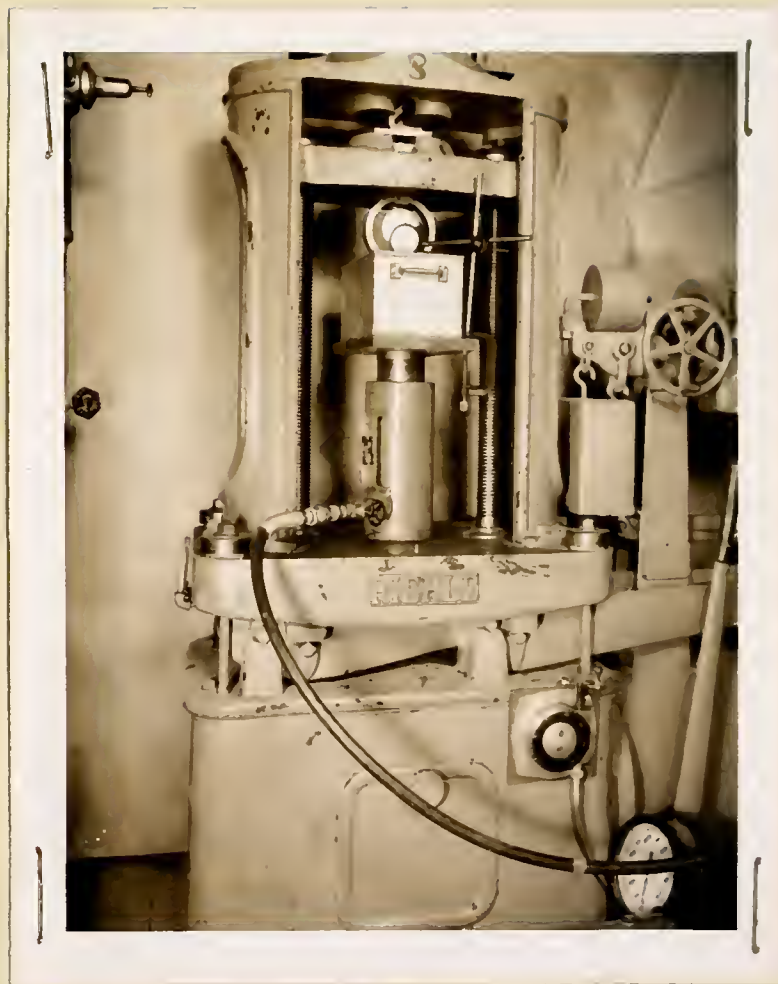
Figure 9.   General View of Repeated Load
Apparatus Ready for Test

testing machine. A hydraulic jack was used in the system to obtain the immediate release of load when the desired load on the specimen was reached. The deformation was measured directly from the top of the specimen (see Figure 10). The compression load was measured by a proving ring which was attached to the movable head of the testing machine. To take up the load of the movable head on the screws, an adjustable spring was used. The control of the rate of deformation and temperature was previously discussed in the Unconfined Compression Test section.

In the repeated load sequence, the rate of deformation, temperature, and applied stress was varied, and the number of load repetitions necessary to cause failure was determined. Figure 11 gives a general representation of three cycles of load repetitions. Deformation measurements were taken so that the elastic rebound and permanent deformation could be determined. The dotted portion of the curve indicates that the period of time between loadings was an undetermined variable. A sufficient period of time was allowed between load applications in order to permit most of the retarded rebound to take place.

Before testing, the specimens were maintained at the test temperature for 30 minutes. The detailed procedure for testing was as follows:

1. The hydraulic jack was centered in the testing machine. The movable head of the testing machine was at the top of the screws. The specimen in the water bath was centered on the hydraulic jack. The ram of the jack was raised (approximately three inches) until the top of the specimen touched the proving ring base.

Figure 10. Close-Up of Repeated Load Apparatus Minus Water Bath, Showing Load and Deformation System

GENERAL RELATIONSHIP BETWEEN DEFORMATION AND TIME
FOR REPEATED LOAD CYCLES
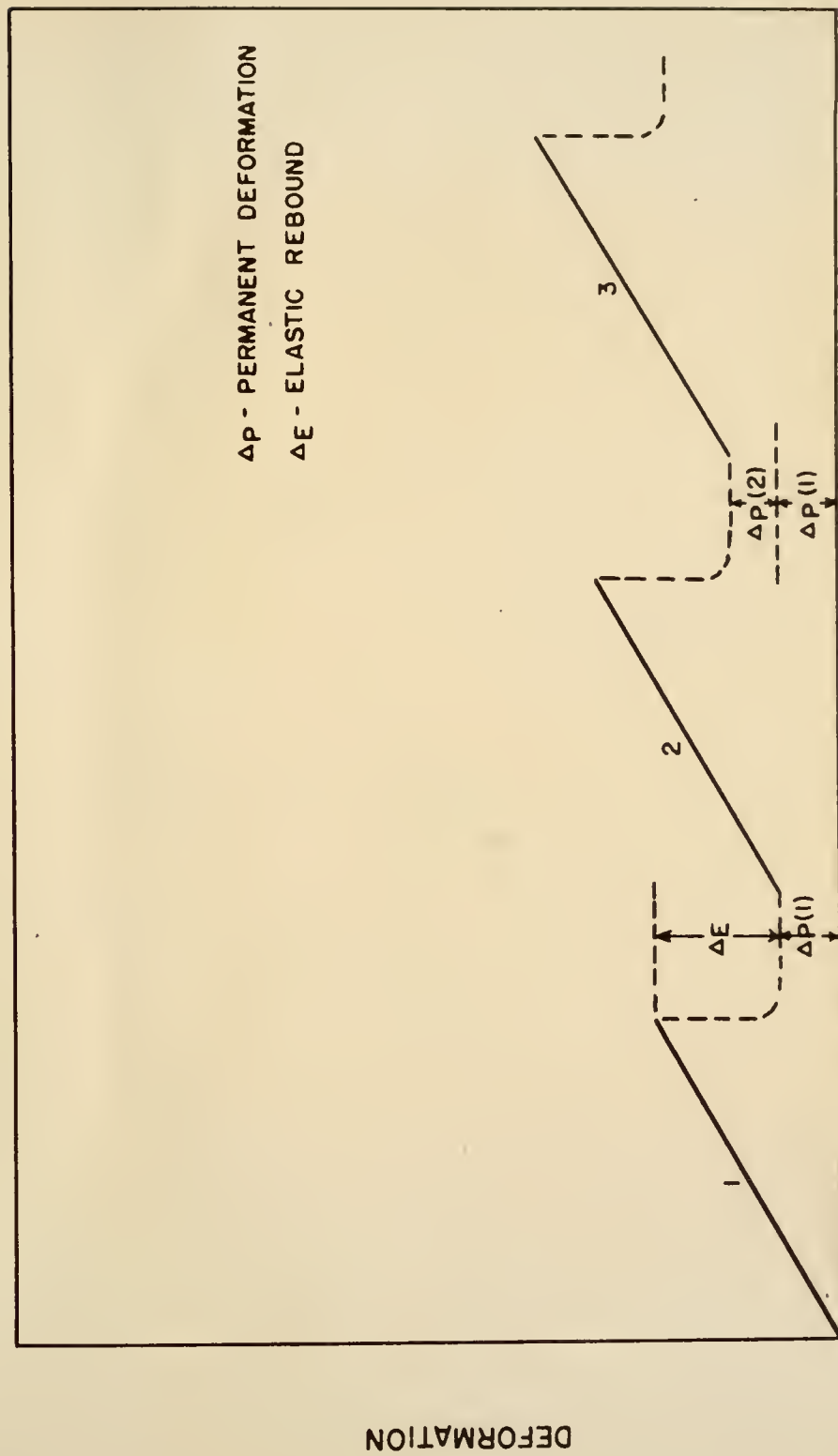
ΔP - PERMANENT DEFORMATION
ΔE - ELASTIC REBOUND

TIME

FIGURE II

DEFORMATION

2. An Ames dial was placed such that measurements could be taken from the top of the specimen. This permitted the operator to control the rate of deformation during loading, to obtain the total deformation at the desired load, and to obtain the rebound and the permanent deformation after the desired load was released.

3. The variable speed drive was set to the desired rate of deformation. When the desired load was reached, the deformation dial reading was recorded and the valve in the hydraulic jack was opened and the testing machine stopped. The specimen was allowed to rebound. When the rebound was essentially completed the deformation dial reading was again recorded. For a sample data sheet, see Figure 12. This permitted one to calculate the elastic rebound and the permanent deformation. The ram of the jack was again raised and the test repeated.

Confined Repeated Load Test

The confined, repeated load test was performed to obtain results necessary to establish a relationship among the following variables: applied stress, temperature, rate of deformation, and number of load applications necessary to reach some failure criterion at varying degrees of lateral confinement.

The evolution of the failure criterion and the method of conducting the repeated load test was discussed previously in the section dealing with the Unconfined Repeated Load Test.

Since this test sequence consisted of two distinct parts, it was felt that the discussion of apparatus and procedure could best be covered in two sub-sections: (a) triaxial compression test, and (b) confined, repeated load test.

DATA SHEET

Repeated Load Test - Bituminous-Aggregate Mixtures

Specimen No. ____I - 15____   Rate of Deformation __0.002 in./min.__

Date Tested __12-5-55____   Temperature _____40°F._____

By _____L.W. & R.P._____   % Ultimate _____50_____

Proving Ring ____4000 lb.____   Dial Reading _____125_____

Lateral Pressure __O psi____   Applied Stress _____210 psi_____

Cycle No.	Dial Reading (unload)	Total Permanent Deformation	Dial Reading (load)	Rebound
0	.0500	—	.0242	—
1	.0317	.0183	.0189	.0075
2	.0272	.0228	.0166	.0083
3	.0239	.0261	.0148	.0073
4	.0219	.0281	.0134	.0071
5	.0203	.0297	.0117	.0069
6	.0198	.0302	.0106	.0081
7	.0189	.0311	.0096	.0083
8	.0179	.0321	.0088	.0083
9	.0169	.0331		.0081

FIGURE 12

Triaxial Compression Test

The apparatus used in the triaxial compression test are as follows: loading frame with Graham variable speed drive, triaxial cell, rubber membrane, Hobart Brothers Air Compressor, a DeVilbiss air pressure regulator, 500 watt immersion heater, a Power-Stir electric stirrer, Voltbox voltage regulator, and a water bath.

The triaxial cell was constructed in the Engineering Science machine shop. A schematic diagram of the cell is shown in Figure 13 and the actual test set-up is presented in Figure 14. The load to the specimen was applied through a vertical screw driven by a variable speed motor. A proving ring was used to measure the load. The control of the rate of deformation and temperature was covered in the section dealing with the unconfined compression test.

Before testing the specimens were maintained at the test temperature for 30 minutes. The detailed procedure for testing was as follows:

1. A rubber membrane was rolled up and fitted over the lower porous disk. The specimen was placed on the lower porous disk and a membrane rolled up over the specimen and onto the upper porous disk. Another membrane was placed over the upper porous disk and rolled down over the specimen. The fit was such that an airtight seal resulted. The pressure cell was then lowered over the specimen. Care was taken to insure that the water level was sufficiently high to cover the upper testing head. Figure 15 shows the specimen in position inside the lucite cylinder.

2. The loading piston was fitted through the top cover plate and the ball-footed end placed in the socket in the upper testing head. The top cover plate was then set down on the threaded connecting bolts.

DETAILS OF TRIAXIAL CELL
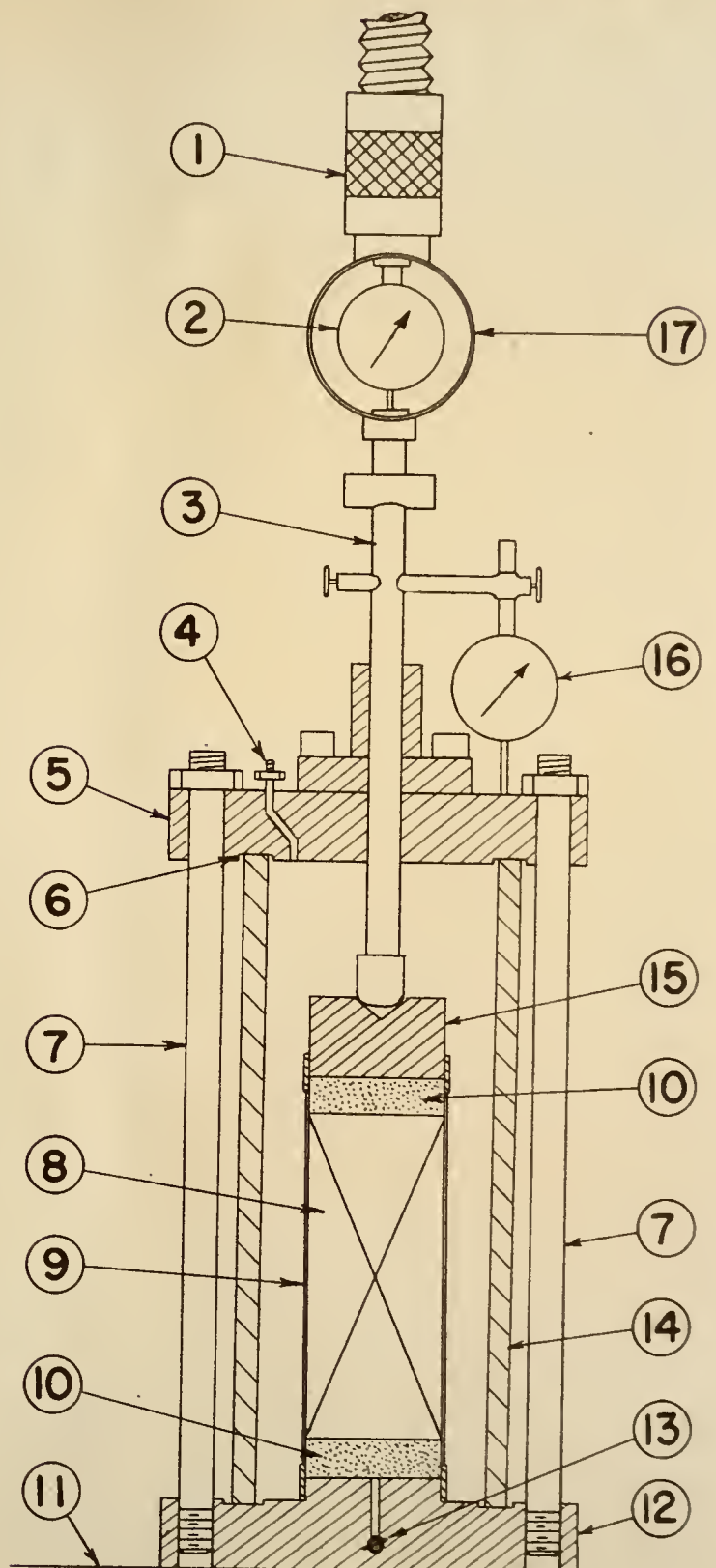
FIGURE 13

Key to Figure 13

1. Turnbuckle for applying seating load.

2. Proving ring deflection dial.

3. Loading piston.

4. Compressed air inlet.

5. Cover plate.

6. Recess for cylinder.

7. Connecting bolts.

8. Test specimen.

9. Rubber membrane.

10. Porous stone.

11. Table of testing machine.

12. Base plate.

13. Drainage line.

14. Wall of triaxial pressure cell.

15. Upper testing head.

16. Strain dial.

17. Proving ring.
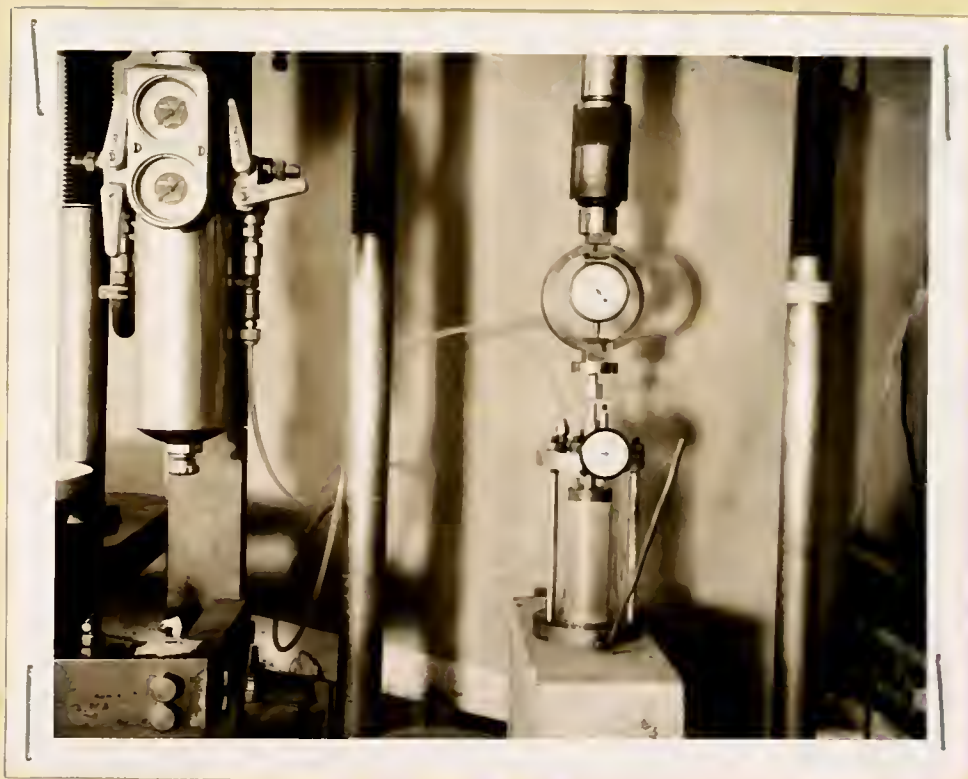
Figure 14.   General View of Triaxial Cell
Ready for Test Minus Water Bath

Figure 15.   Triaxial Cell Without Top Cover

The loading piston was then checked for alignment. The nuts were placed on the three connecting bolts and the apparatus fastened together. The deformation dial was attached to the cell.

3. The triaxial cell was connected with the pressure regulator and then the water bath with the cell was centered in the loading frame. A small seating load, approximately 10 pounds, was applied to the specimen. The deformation dial was then zeroed.

4. The air compressor was previously turned on and the main line pressure allowed to build up. By adjusting the air pressure regulator, the desired confining pressure was developed in the cell.

5. The desired rate of deformation was set on the Graham drive. As loading progressed, load and deformation dial readings were taken and recorded at fixed levels of deformation (see Figure 8 for a typical data sheet). The loading was applied until the maximum value was obtained. The rate of deformation was continually checked.

6. When the maximum load was obtained, the motor was shut off and the air released from the cell. The direction of the movement of the shaft was reversed and the load released. The apparatus was taken apart and made ready for another test.

Confined Repeated Load Test

The apparatus used in this series was a combination of the apparatus required in the unconfined, repeated load test and the triaxial compression test and included the following: loading frame with Graham variable speed drive, triaxial cell, rubber membrane, Hobart Brothers Air Compressor, a DeVilbiss air pressure regulator, Black Hawk hydraulic

jack (50 ton capacity), 500 watt immersion heater, Voltbox voltage regu-lator, Power-Stir electric stirrer, and a water bath.

The detailed procedure followed exactly parts 1 and 2 in the sec-tion Triaxial Compression Test.

3. The water bath with the triaxial cell was placed on the hy-draulic jack. The entire assembly was centered in the loading frame. The ram of the jack was raised three to four inches until the top of the loading piston was in contact with the base of the proving ring.

4. An Ames dial was fixed to the loading piston such that measure-ments could be taken from the top of the specimen. This permitted the operator to control the rate of deformation during loading, to obtain the total deformation at the desired load, and to obtain the rebound and the permanent deformation after the desired load was released.

5. The triaxial cell was connected with the pressure regulator. A small seating load, approximately 10 pounds, was applied to the speci-men. The deformation dial was then zeroed. The test set-up is shown in Figure 16.

6. The air compressor was previously turned on and the main line pressure allowed to build up. By adjusting the air pressure regulator, the desired confining pressure was introduced into the cell.

7. The variable speed drive was set to the desired rate of defor-mation and the test started. When the desired load was reached, the deformation reading was taken, the valve in the hydraulic jack was opened, the air pressure in the cell was released, and the drive motor stopped. The specimen was allowed to rebound. When the rebound was essentially completed the deformation dial reading was again recorded.
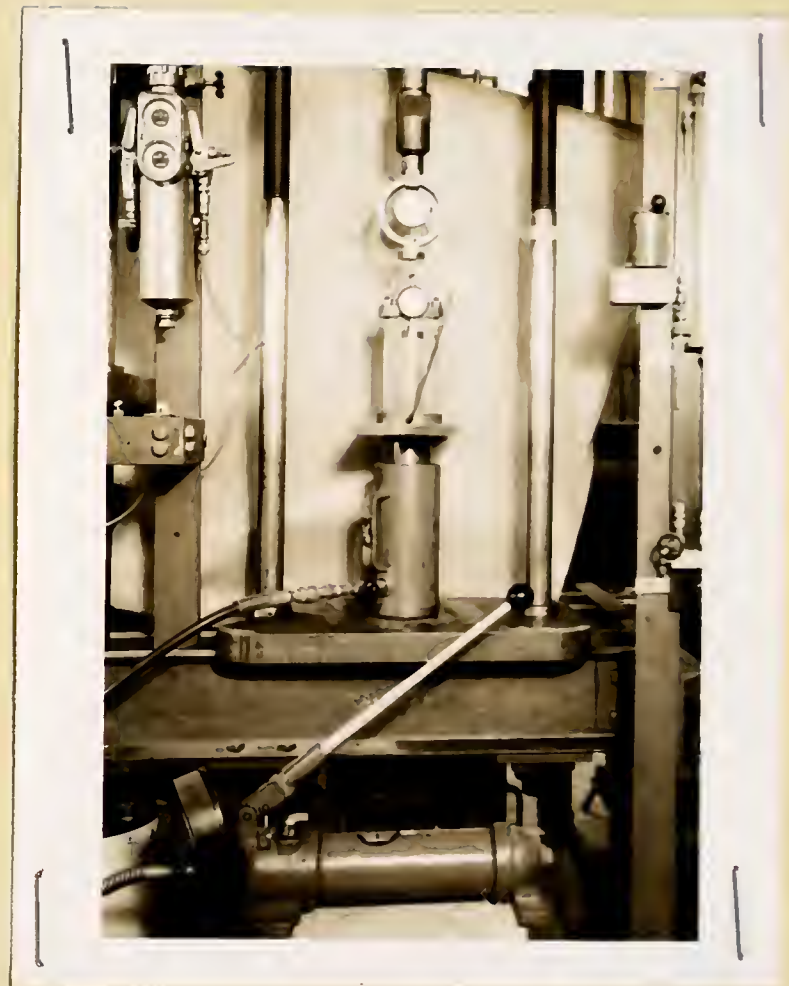
Figure 16.   General View of Confined Repeated
               Load Apparatus Ready for Test

For a sample data sheet, see Figure 12. This permitted one to calculate the elastic rebound and the accumulated permanent deformation. The ram of the jack was again raised and the test repeated.

Static Load Test

The static load test was devised to obtain the relationship among the following variables: applied stress, temperature, total deformation, and time. The inclusion of the static load test was for the purpose of enabling one to make a comparison of the adequacy of a mixture based upon the repeated load series or the static load series.

The apparatus used in this test series was as follows: Coiltest consolidation apparatus, 500 watt immersion heater, Voltbox voltage regulator, Power-Stir electric stirrer, a stop watch, and a water bath.

Before testing the specimens were maintained at test temperature for 30 minutes. The detailed procedure for testing was as follows:

1. The water bath containing the test specimen was centered in the consolidation frame. The Ames dials were attached so that the deformation during loading and the rebound after the load was removed could be observed. The test set-up is shown in Figure 17.

2. A small seating load was applied to the specimen (approximately four pounds). The deformation dials were zeroed. The static load was placed on the specimen at time zero and the stop watch started. The deformation was recorded at five seconds, 20 seconds, 40 seconds, 60 seconds, 100 seconds and every additional 100 seconds until the test was halted and the load removed. For a sample data sheet see Figure 18.
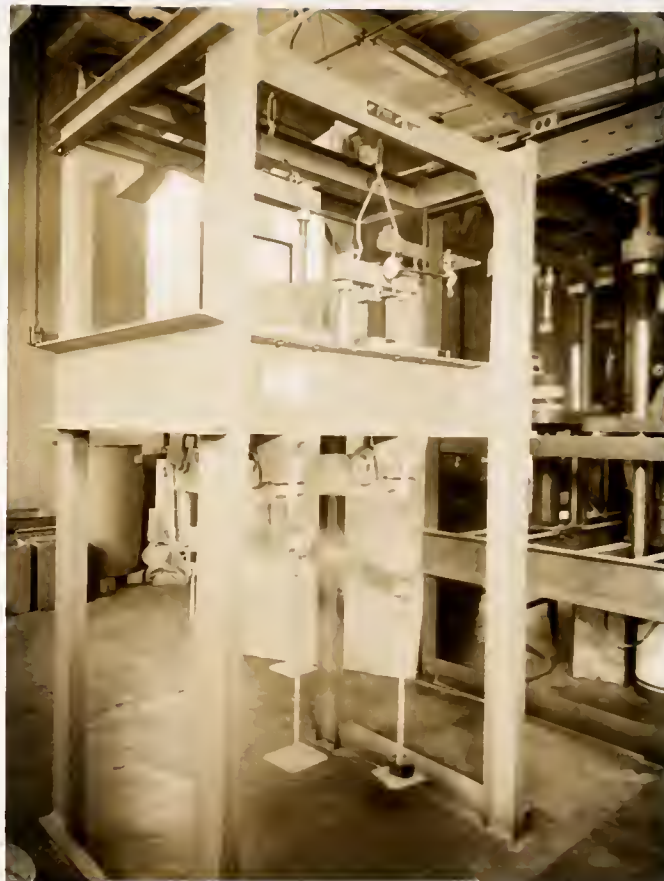
Figure 17.   General View of Static Load
             Apparatus Ready for Test.

DATA SHEET

Static Load Test - Bituminous-Aggregate Mixtures

Specimen No. _____ I-77 _____   Temperature _____ 40°F. _____

Date Tested _____ 3-8-55 _____   Static Load _____ 200 psi _____

By _____ L.W. & R.P. _____

Time Sec.	Dial Readings 1	2	Deformation (in.) 1	2	Deformation (in.) Average
0	.499	.500	—	—	—
5	.490	.489	.009	.011	.010
10	.489	.488	.010	.012	.011
20	.488	.486	.011	.014	.012
40	.487	.485	.012	.015	.014
60	.486	.484	.013	.016	.015
100	.484	.482	.015	.018	.017
200	.480	.478	.019	.022	.020
300	.476	.473	.023	.027	.025
400	.473	.470	.026	.030	.028
500	.470	.467	.029	.033	.031
600	.468	.465	.031	.035	.033
700	.466	.462	.033	.038	.036
800	.465	.461	.034	.039	.037
900	.463	.459	.036	.041	.039
1000	.462	.458	.037	.042	.040
1100	.461	.457	.038	.043	.041
1200	.460	.456	.039	.044	.042
1200, unload	.473	.469	.026	.031	.028
1205	.474	.471	.025	.029	.027
1210	.474	.471	.025	.029	.027
1220	.475	.472	.024	.028	.026
1240	.476	.473	.023	.027	.025
1260	.476	.474	.023	.026	.025
1300	.477	.475	.022	.025	.024
1400	.478	.476	.021	.024	.023
1500	.478	.476	.021	.024	.023

FIGURE 18

3. The criterion for halting the test was the ability or the inability of the specimen to withstand the applied test load. A specimen withstood the applied test load if the deformation increased only 0.001 inches per 100 seconds for 400 seconds. A specimen failed to withstand the applied test load when the deformation rate per unit time increased rather than decreased. When the deformation rate per unit time increased, it was noted that complete failure of the test specimen was imminent.

4. The specimen rebound upon removal of the test load was followed and deformation values recorded at fixed time intervals until essentially all of the rebound had occurred.

RESULTS

In this investigation, strength-deformation data of bituminous mixtures were obtained by means of conventional compression tests and by means of repeated load tests. These tests are described in the section dealing with Apparatus and Procedure. The aggregate type and gradation are discussed in the Materials section. The two sand-asphalt mixtures designated as Mixture I and Mixture II differed only in the physical character of the material passing the No. 200 sieve. The minus 200 material used in Mixture II was somewhat coarser than the minus 200 material used in Mixture I.

The type of test, the mixture tested, and the variables studied for each test series are outlined as follows:

I. Compressive Strength Tests

    A. Unconfined Compression Tests

        1. Sand-Asphalt Mixture I

            Variables: (a) temperature

                      (b) rate of deformation

        2. Sand-Asphalt Mixture II

            Variables: (a) temperature

                      (b) rate of deformation

        3. Indiana AH Type B Surface Course Mixture

            Variables: (a) temperature

                      (b) rate of deformation

B. Confined Compression Tests

    1. Sand-Asphalt Mixture II

        Variables: (a) confining pressure

                (b) temperature

                (c) rate of deformation

C. Static Unconfined Load Tests

    1. Sand-Asphalt Mixture II

        Variables: (a) applied stress

                (b) temperature

II. Repeated Load Tests

A. Unconfined, Repeated Load Test

    1. Sand-Asphalt Mixture II

        Variables: (a) applied stress

                (b) temperature

                (c) rate of deformation

B. Confined, Repeated Load Test

    1. Sand-Asphalt Mixture II

        Variables: (a) confining pressure

                (b) applied stress

                (c) temperature

                (d) rate of deformation

The results in both tabular and graphical form are presented following the above outline.

Unconfined Compression Tests

In the unconfined compression series of tests the effect of temperature and rate of deformation upon the maximum unconfined compressive stress for several mixtures was determined. Table 5 shows the results of five rates of deformation: 0.002, 0.02, 0.2, 2.0, and 8.65 in./min. and three temperatures: 40°, 100°, and 140°F upon the maximum unconfined compressive stress of Mixture I. The results from Table 5 are depicted graphically in Figures 19 and 20. In Figure 19 the maximum unconfined compressive stress is plotted versus the rate of deformation for the various temperatures. Figure 20 shows the maximum unconfined compressive stress plotted against the temperature for the various rates of deformation.

Table 6 presents the results of three rates of deformation: 0.002, 0.02, and 0.2 in./min. and three temperatures: 40°, 100°, and 140°F upon the maximum unconfined compressive stress of Mixture II. The results from Table 6 are shown graphically in Figure 21 where the maximum unconfined compressive stress is plotted versus the rate of deformation for various temperatures.

The family of curves in Figure 19 was expressed in a mathematical model. This model and the methods used to establish the model are discussed in the section dealing with Regression Analysis. This general model was then used as a prediction equation. From a limited amount of laboratory test data on Mixture II, the parameters of the model were evaluated and the general model was then used as a prediction equation to estimate strength values for various conditions of temperature and

rate of deformation. To check the predictions, tests were made to obtain sufficient data to get a comparison between predicted values and observed values. The predicted values and observed values are denoted in Tables 6 and 7 and in Figures 21 and 22.

Table 7 shows the results of three rates of deformation: 0.002, 0.02, and 0.2 in./min. and three temperatures: 40°, 100°, and $140^{\circ}F$ upon the maximum unconfined compressive stress for the Indiana AH Type B Surface Course Mixture. The results from Table 7 are depicted graphically in Figure 22 where the maximum unconfined compressive stress is plotted versus the rate of deformation for various temperatures.

Table 5

Unconfined Compression Test Results for Mixture I

Temperature °F	Rate of Deformation in./min.				
	0.002	0.02	0.2	2.0	8.65
	Maximum Compressive Stress (psi)				
40	445	660	1045	1705	2015
100	29	71	128	257	350
140	10	23	41	86	116

RELATIONSHIP BETWEEN RATE OF LOADING AND
MAXIMUM COMPRESSIVE STRESS AT VARIOUS TEMPERATURES

MIXTURE I
UNCONFINED TEST

LEGEND
△ – 40°F
□ – 100°F
○ – 140°F

RATE OF LOADING – IN./MIN (LOG SCALE)

MAXIMUM COMPRESSIVE STRESS – PSI
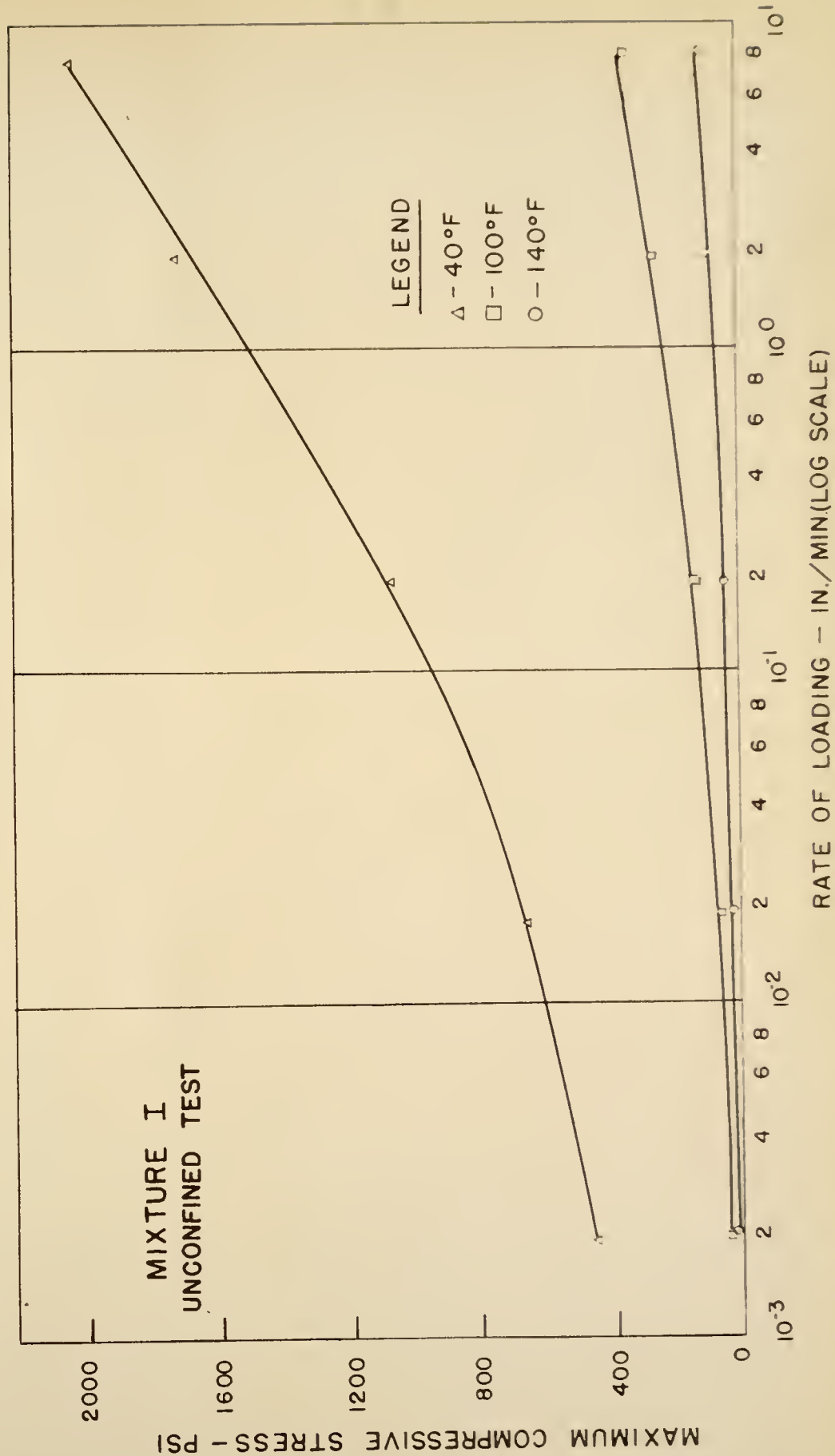
FIGURE 19

RELATIONSHIP BETWEEN MAXIMUM COMPRESSIVE STRESS AND TEMPERATURE AT VARIOUS RATES OF DEFORMATION
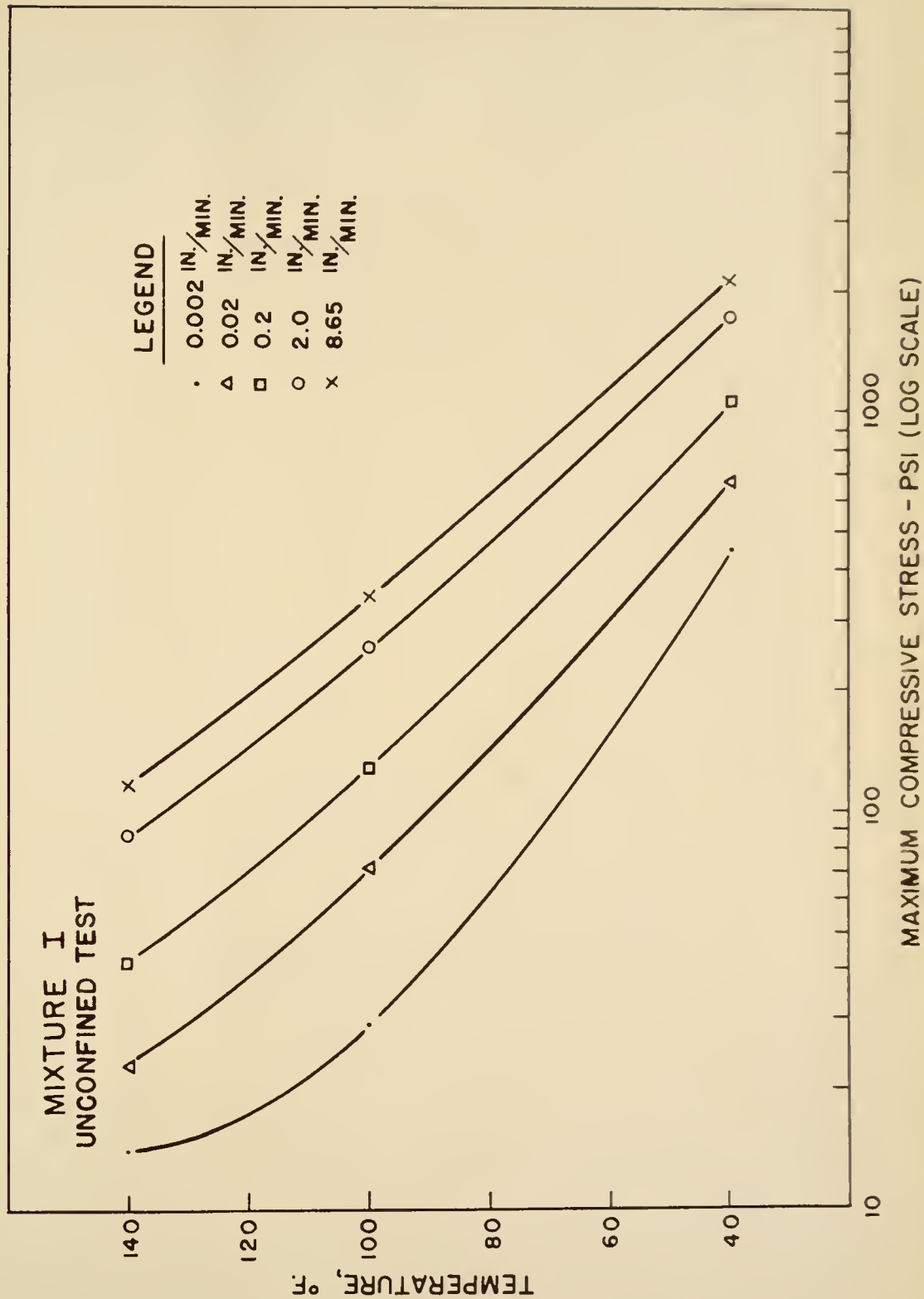
FIGURE 20

Table 6

Unconfined Compression Test Results for Mixture II

Temperature $^\circ$F	Rate of Deformation - in./min.		
	0.002	0.02	0.2
	Maximum Compressive Stress (psi)		
40	420^1	649^2 464^3	1035^1
100	29^2 21^3	50^2 46^3	91^2 100^3
140	8^1	14^2 16^3	26^1

[1] Values used in establishing mathematical relationship for Mixture II.

[2] Calculated values.

[3] Observed test values taken after the relationship for Mixture II was established.

RELATIONSHIP BETWEEN RATE OF DEFORMATION AND
MAXIMUM COMPRESSIVE STRESS AT VARIOUS TEMPERATURES
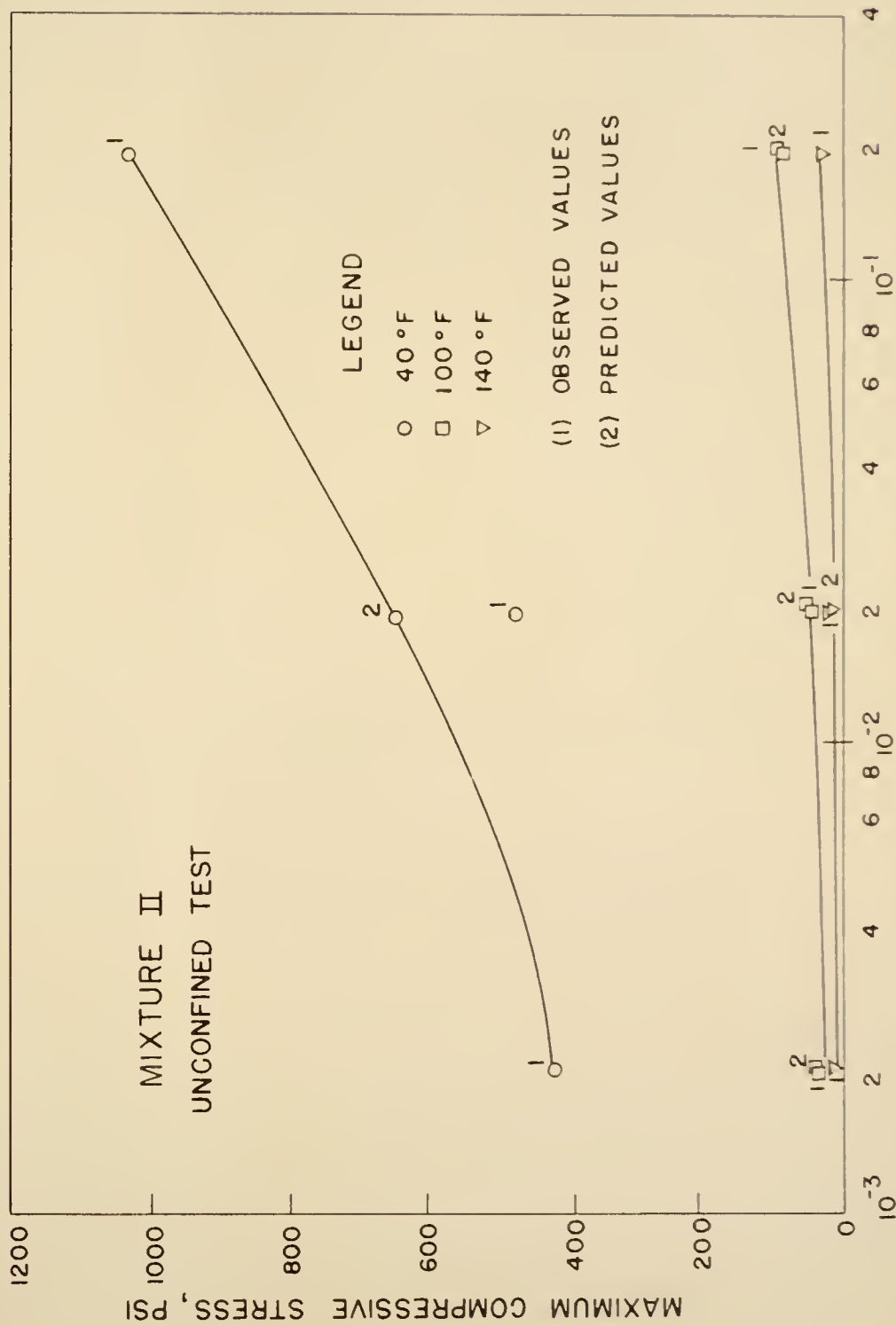
FIGURE 21

Table 7

Unconfined Compression Test Results for Indiana AH Type B Mixture

Temperature $^{\circ}F$	Rate of Deformation - in./min.		
	0.002	0.02	0.2
	Maximum Compressive Stress (psi)		
40	427^{1}	648^{2} 708^{3}	1043^{1}
100	72^{2} 64^{3}	110^{2} 96^{3}	175^{2} 145^{3}
140	32^{1}	45^{2} 34^{3}	70^{1}

[1] Values used in establishing mathematical relationship for Indiana AH Type B Surface Mixture.

[2] Calculated values.

[3] Observed test values taken after the relationship was established.

RELATIONSHIP BETWEEN RATE OF DEFORMATION AND
MAXIMUM COMPRESSIVE STRESS AT VARIOUS TEMPERATURES

INDIANA AH TYPE B SURFACE COURSE
UNCONFINED TEST

LEGEND

○   40 °F
□   100 °F
△   140 °F

(1) OBSERVED VALUES

(2) PREDICTED VALUES

RATE OF DEFORMATION, IN./MIN.(LOG SCALE)
FIGURE 22

MAXIMUM COMPRESSIVE STRESS, PSI
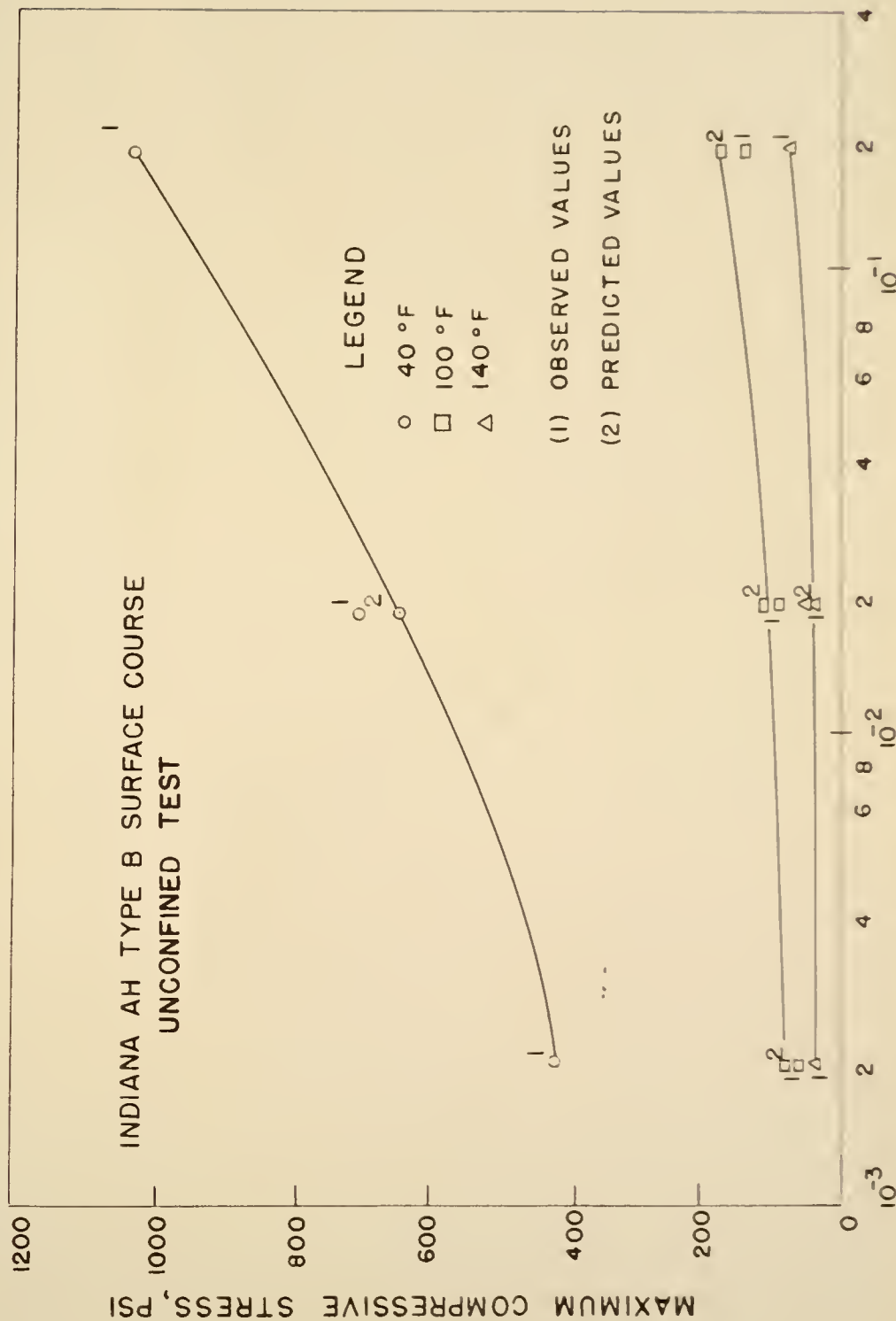
Confined Compression Tests

The series of confined compression tests was performed on the sand-asphalt Mixture II for the purpose of determining the effect of lateral support upon the maximum compressive stress at various temperatures and rates of deformation. The maximum confined compressive stresses at three rates of deformation: 0.002, 0.02, 0.2 in./min. and three temperatures: 40°, 100°, 140°F, and two confining pressures: 15 and 30 psi are shown in Table 8 and are depicted graphically in Figures 23 and 24. In each figure the maximum compressive stress at one confining pressure is plotted against rate of deformation for various temperatures.

From a limited amount of laboratory test data on Mixture II, the parameters of the regression equation for the confined cases were evaluated and the equation was then used as a prediction equation to estimate strength values for various conditions of temperature and rate of deformation. To check the predictions, tests were made to obtain sufficient data to get a comparison between predicted values and observed values. The predicted values and observed values are shown in Figures 23 and 24.

Table 8

Confined Compression Test Results for Mixture II

Temperature °F	Lateral Pressure - psi	Rate of Deformation - in./min.		
		0.002	0.02	0.2
		Maximum Compressive Stress (psi)		
40	15	390	609	1080
	30	459	680	1100
100	15	103	114	177
	30	168	195	222
140	15	94	96	104
	30	157	170	173

RELATIONSHIP BETWEEN RATE OF DEFORMATION AND MAXIMUM COMPRESSIVE STRESS AT VARIOUS TEMPERATURES
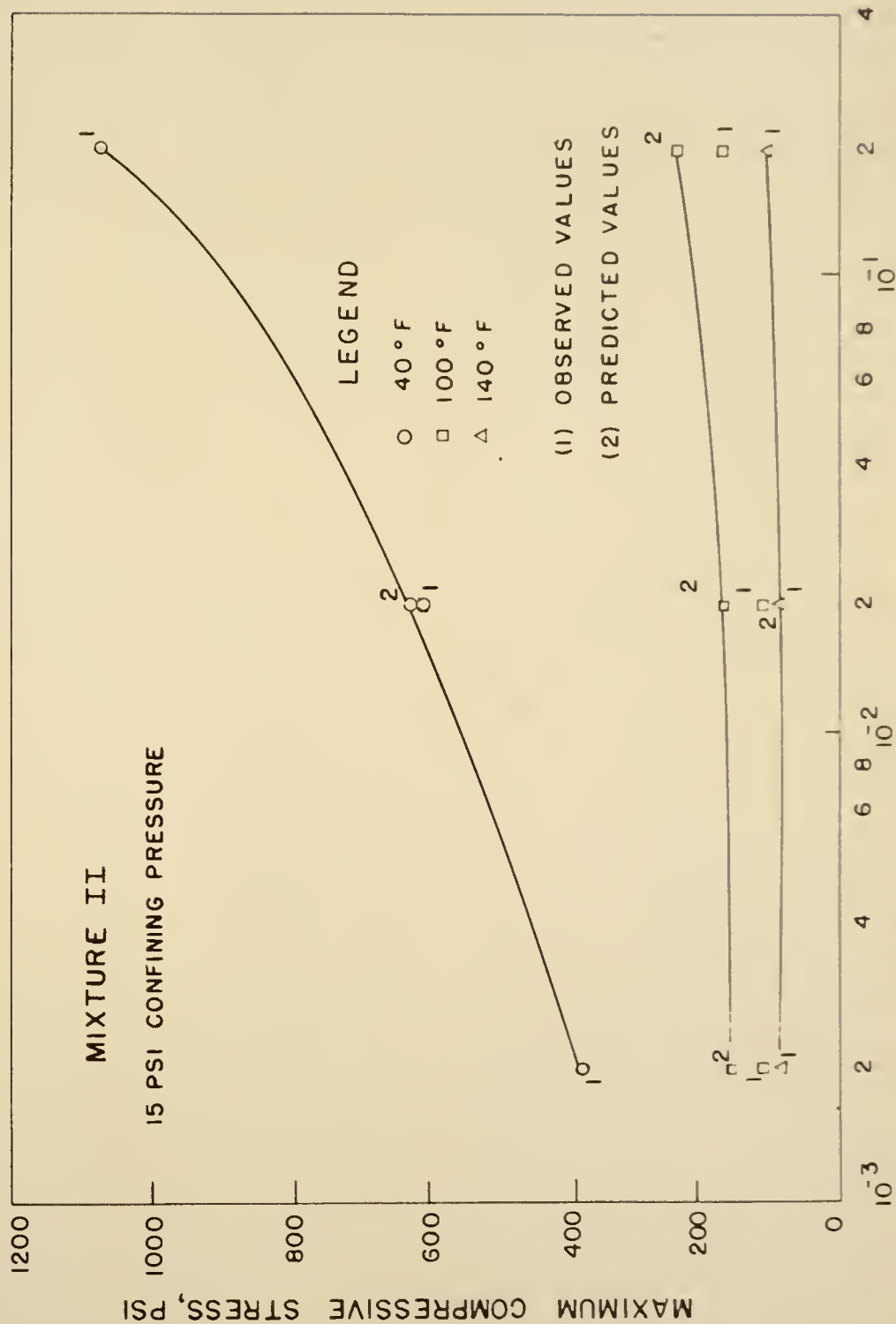
FIGURE 23

RELATIONSHIP BETWEEN RATE OF DEFORMATION AND
MAXIMUM COMPRESSIVE STRESS AT VARIOUS TEMPERATURES

MIXTURE II
30 PSI CONFINING PRESSURE

LEGEND

○ 40°F
□ 100°F
△ 140°F

(1) OBSERVED VALUES

(2) PREDICTED VALUES

MAXIMUM COMPRESSIVE STRESS, PSI
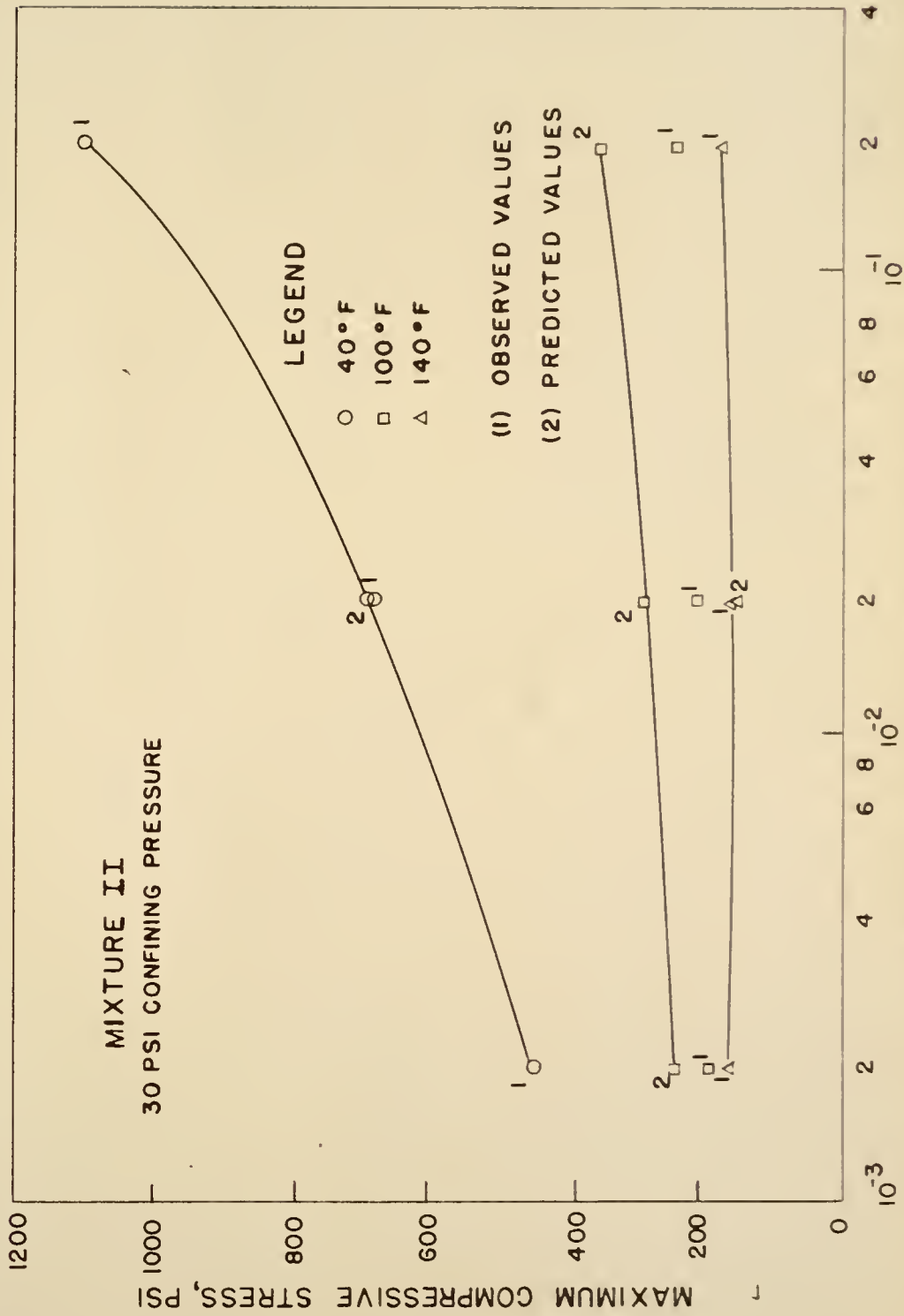
RATE OF DEFORMATION IN./MIN.(LOG SCALE)

FIGURE 24

Unconfined, Repeated Load Tests

In this part of the study, an evaluation of the effect of repeated loads upon a specimen tested in unconfined compression was undertaken. These repeated loads were applied at varying rates of deformation and temperatures and at fixed percentages of the maximum compressive strength of the mixture as determined from the unconfined compression test. The total deformation and amount of permanent deformation for each load cycle was measured. A sufficient period of time was allowed between load applications in order to permit most of the retarded rebound to take place. The results of this study are presented graphically in Figures 25 through 33, where the permanent deformation is plotted against the number of load repetitions for different applied stresses at various rates of deformation and temperature.

The plot of permanent deformation versus number of load applications starts out as a straight line on a semi-logarithmic plot in all cases. At some stage, dependent upon the applied stress and the number of load applications, the plot deviates sharply from the straight line. The point at which this deviation from linearity occurred was selected as the point of failure and was taken as the failure criterion.

The number of load repetitions necessary to cause failure at the various unconfined test conditions is presented in Table 9 and graphically depicted in Figures 34, 35, and 36. The applied stress is plotted versus the number of load applications necessary to cause failure for the different test conditions in the unconfined test.

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES

FIGURE 25

RELATIONSHIP   BETWEEN   PERMANENT   DEFORMATION

AND   NUMBER   OF   LOAD   REPETITIONS

FOR   VARIOUS   APPLIED   STRESSES

NO. OF LOAD REPETITIONS   (LOG SCALE)

FIGURE 26

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES

FIGURE 27

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES

FIGURE 28

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES

FIGURE 29

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES

NO. OF LOAD REPETITIONS (LOG SCALE)
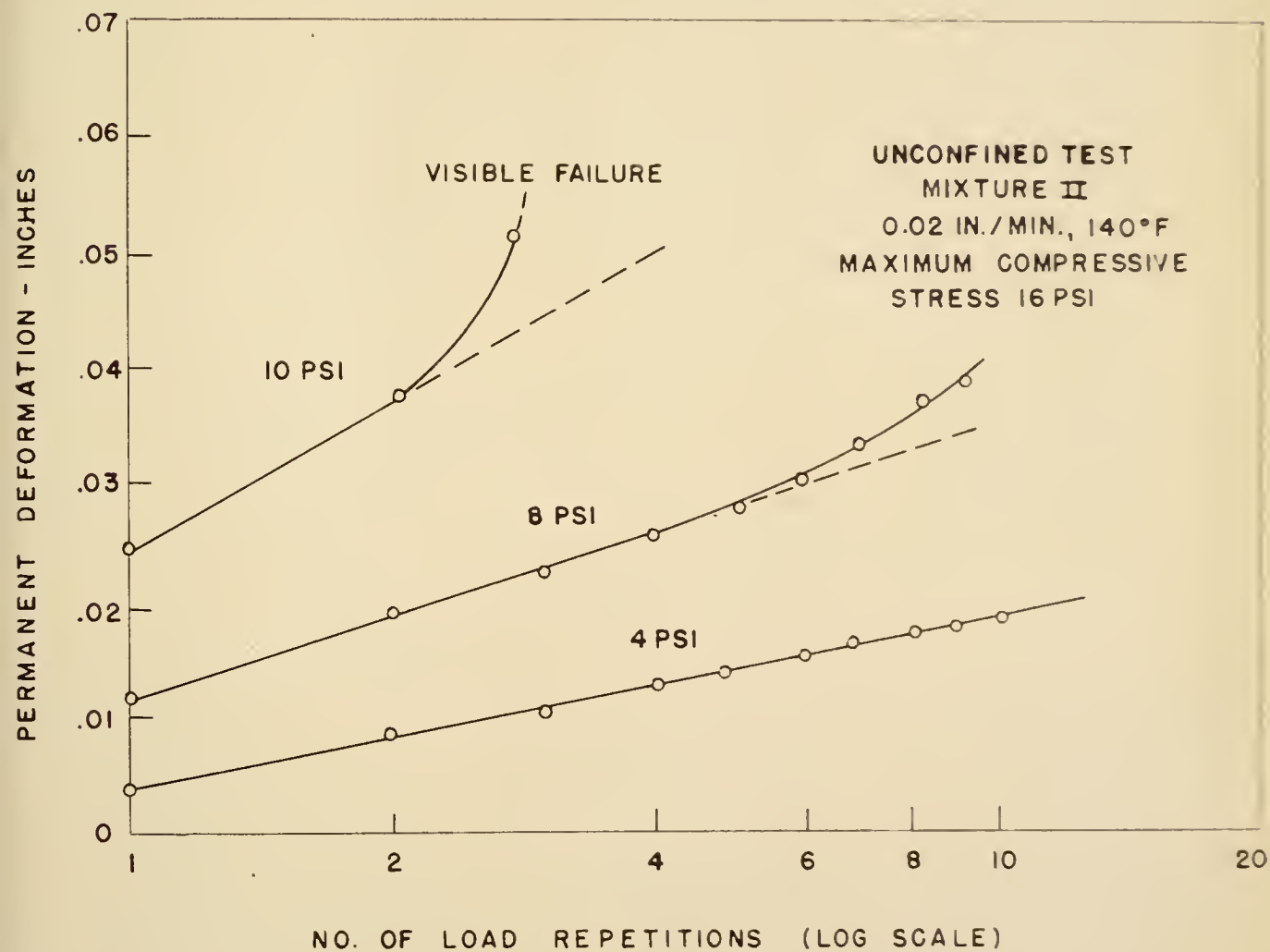
FIGURE 30

RELATIONSHIP BETWEEN PERMANENT DEFORMATION AND NUMBER OF LOAD REPETITIONS FOR VARIOUS APPLIED STRESSES
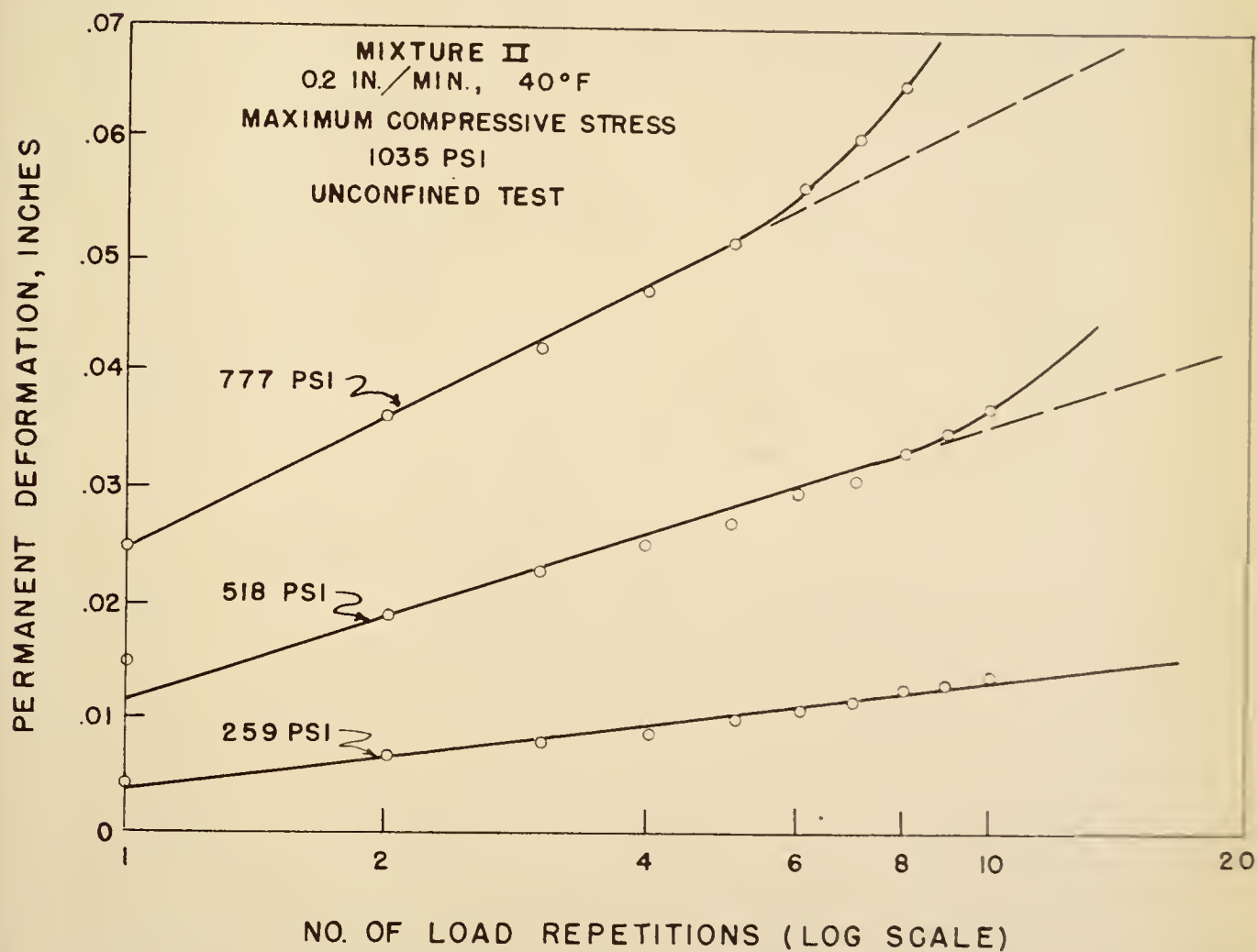
FIGURE 31

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES
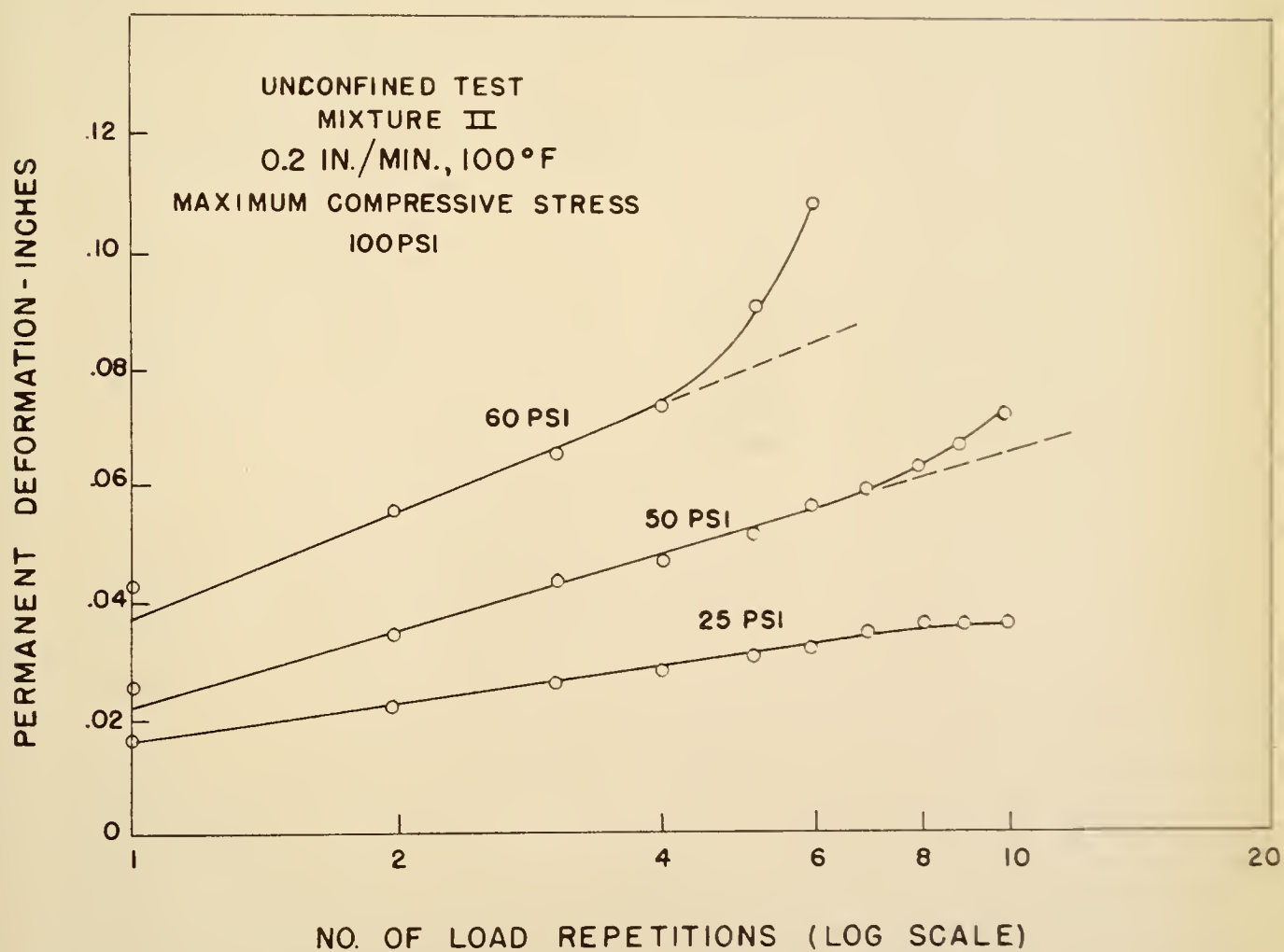
FIGURE 32

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES
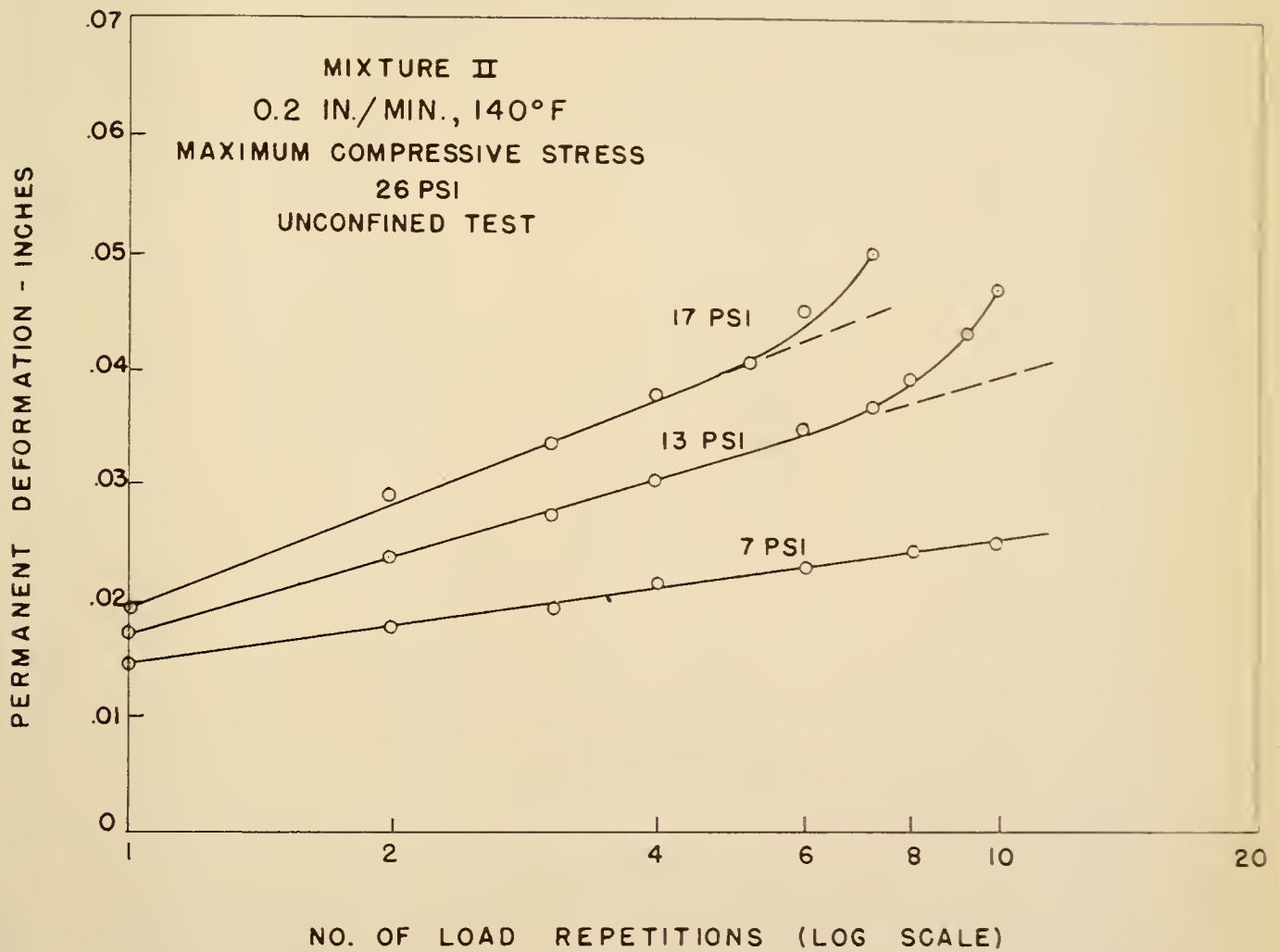
FIGURE 33

APPLIED STRESS — ENDURANCE LIMIT

RELATIONSHIPS AT VARIOUS RATES OF LOADING
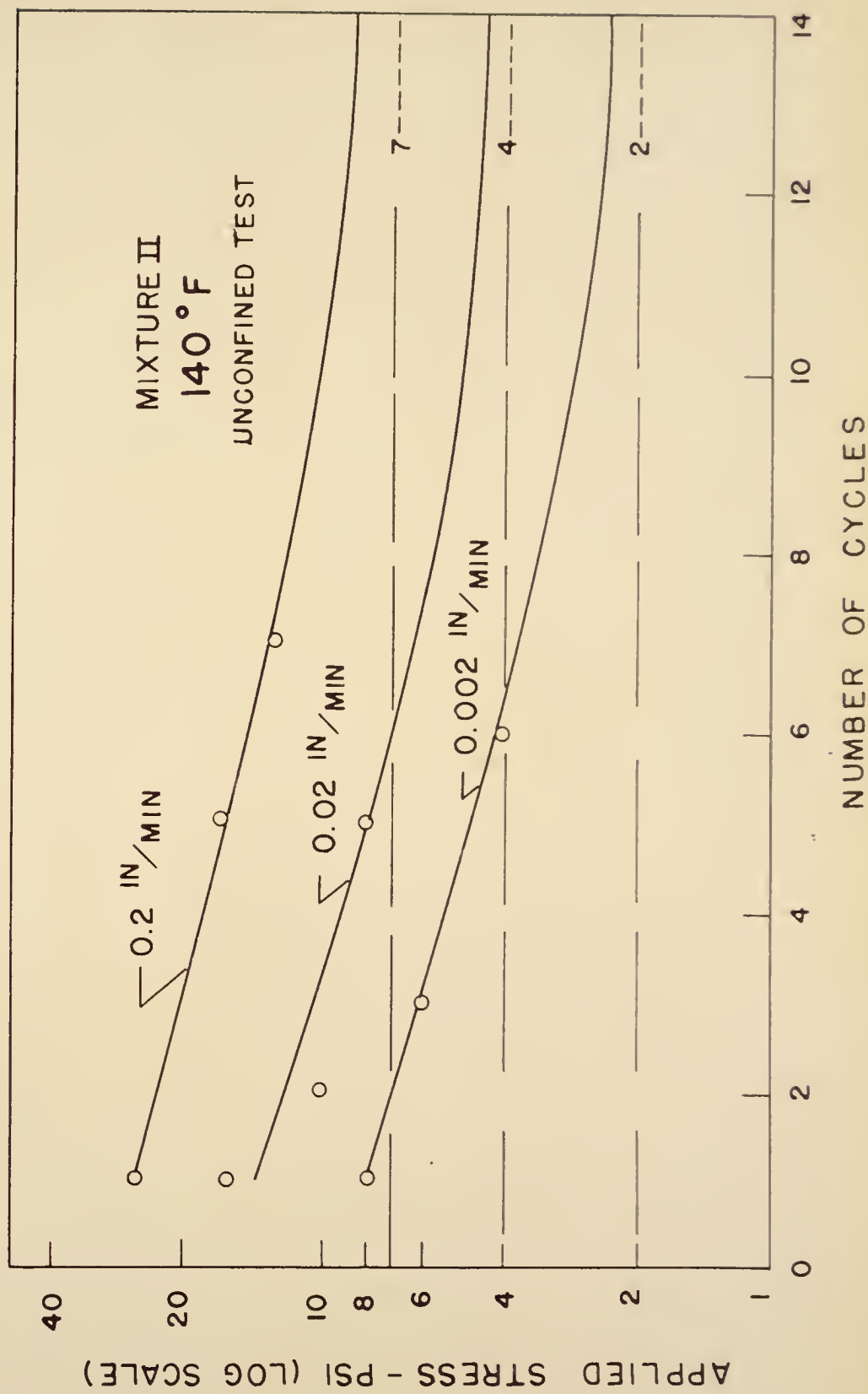
FIGURE 34

APPLIED STRESS — ENDURANCE LIMIT
RELATIONSHIPS AT VARIOUS TEMPERATURES

MIXTURE II
0.2 IN./MIN.
UNCONFINED TEST

NUMBER OF CYCLES

FIGURE 35

89

APPLIED STRESS – ENDURANCE LIMIT
RELATIONSHIPS AT VARIOUS TEMPERATURES
AND RATES OF LOADING

MIXTURE II
UNCONFINED TEST

NUMBER OF CYCLES

FIGURE 36

APPLIED STRESS – PSI (LOG SCALE)

0.2 IN./MIN., 40°F

0.02 IN./MIN., 40°F

0.002 IN./MIN., 40°F

0.2 IN./MIN., 100°F

0.02 IN./MIN., 100°F

0.002 IN./MIN., 100°F

0.2 IN./MIN., 140°F

0.02 IN./MIN., 140°F

0.002 IN./MIN., 140°F
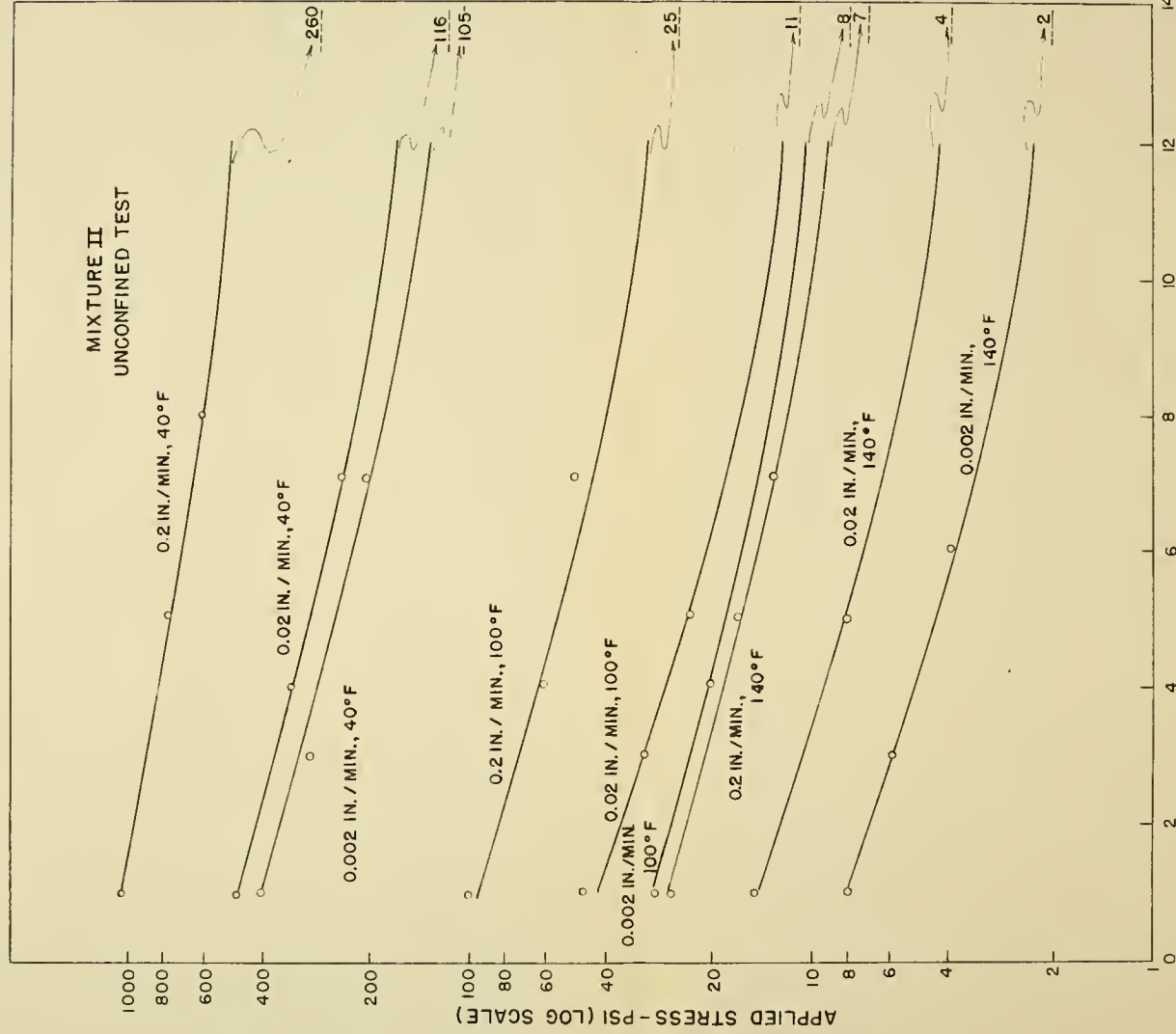
Table 9

Unconfined, Repeated Load Test Results for Mixture II

Temperature °F	Nominal % Max. Comp. Stress	Rate of Deformation - in./min.		
		0.002	0.02	0.2
		Load Repetitions Necessary to Cause Failure[1]		
	75	3	4	5
40	50	7	7	7
	25	**	**	28 -
	75	4^2	3	4^4
100	50	7	5	7
	25	$**^3$	**	**
	75	3	2^4	5^4
140	50	6	5	7
	25	**	**	35 -

[1] For definition of failure, see text.

** Number of load repetitions used during test did not cause failure.

[2] 67% of maximum compressive stress was used.

[3] 33% of maximum compressive stress was used.

[4] 60% of maximum compressive stress was used.

Confined, Repeated Load Tests

The confined, repeated load test series was performed to determine the effect of lateral support upon the relationship of applied stress, number of load applications necessary to reach failure, temperature, and rate of deformation.

The results of this study are shown graphically in Figures 27 through 46. The permanent deformation is again plotted against the number of load applications for different applied stresses at various temperatures and rates of deformation.

The number of load repetitions necessary to cause failure at the various confined test conditions is presented in Table 10 and shown graphically in Figure 47. The applied stress is plotted against the number of load applications necessary to cause failure for the various test conditions.

RELATIONSHIP BETWEEN PERMANENT DEFORMATION AND NUMBER OF LOAD REPETITIONS FOR VARIOUS APPLIED STRESSES
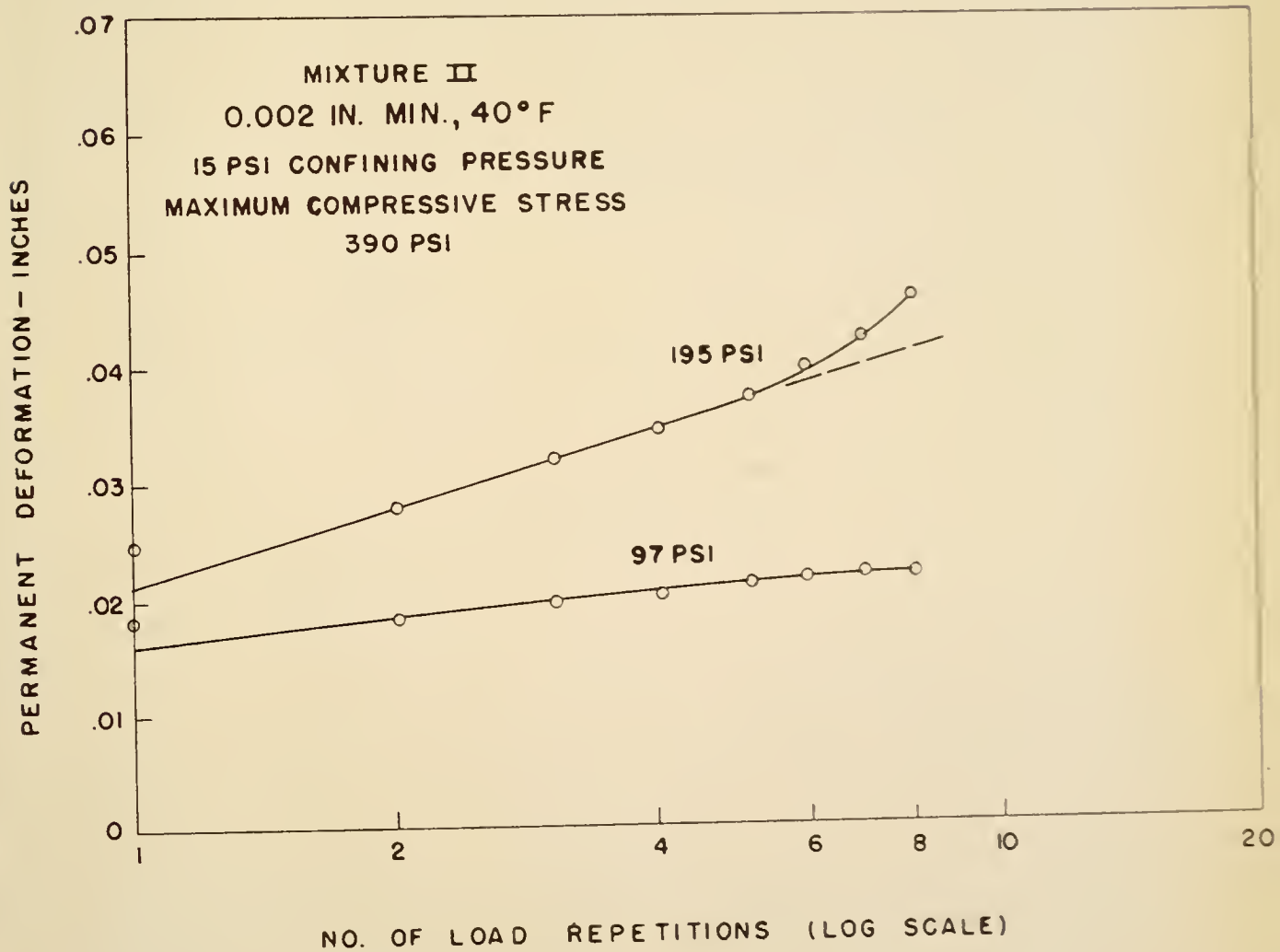
FIGURE 37

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES
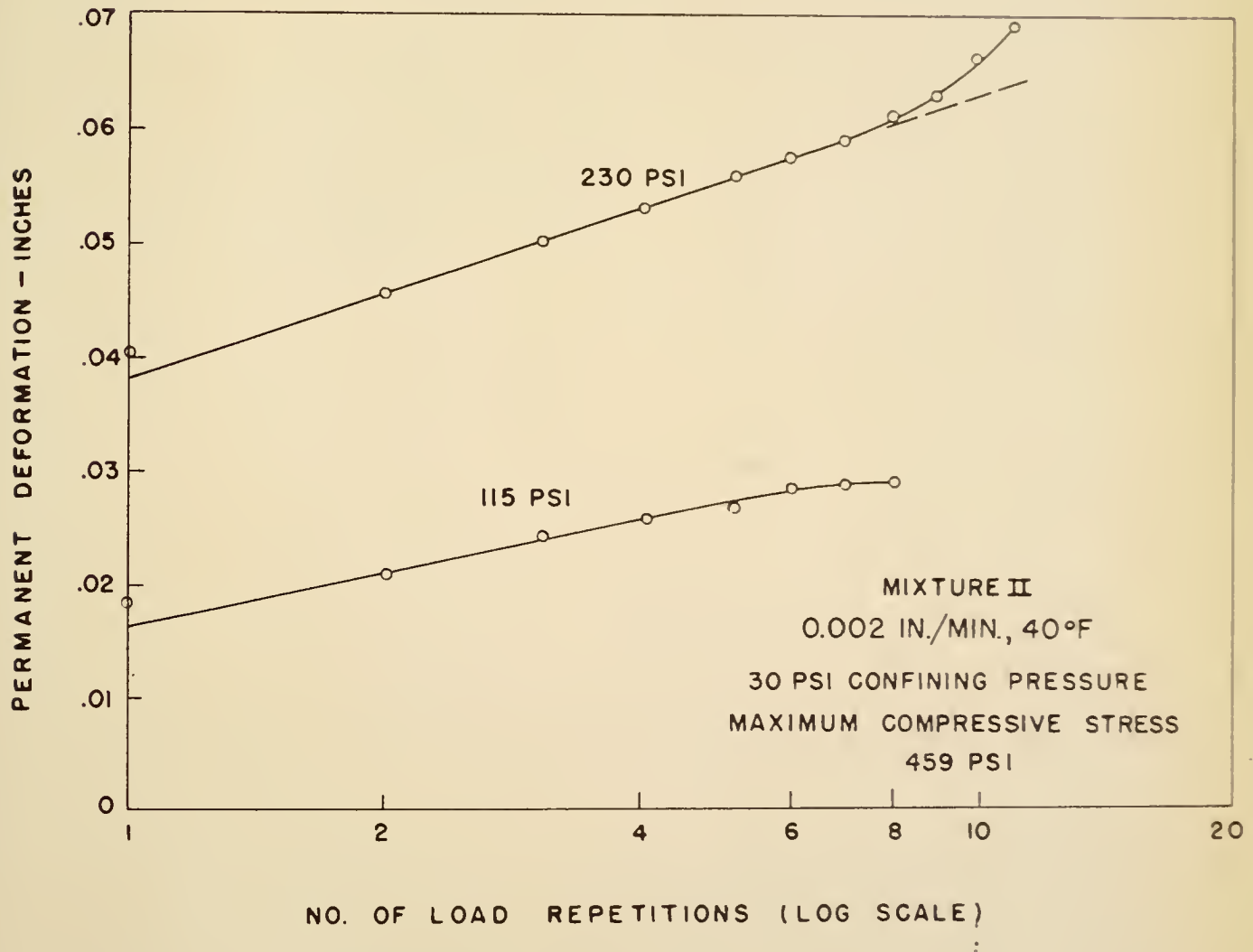
FIGURE 38

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES
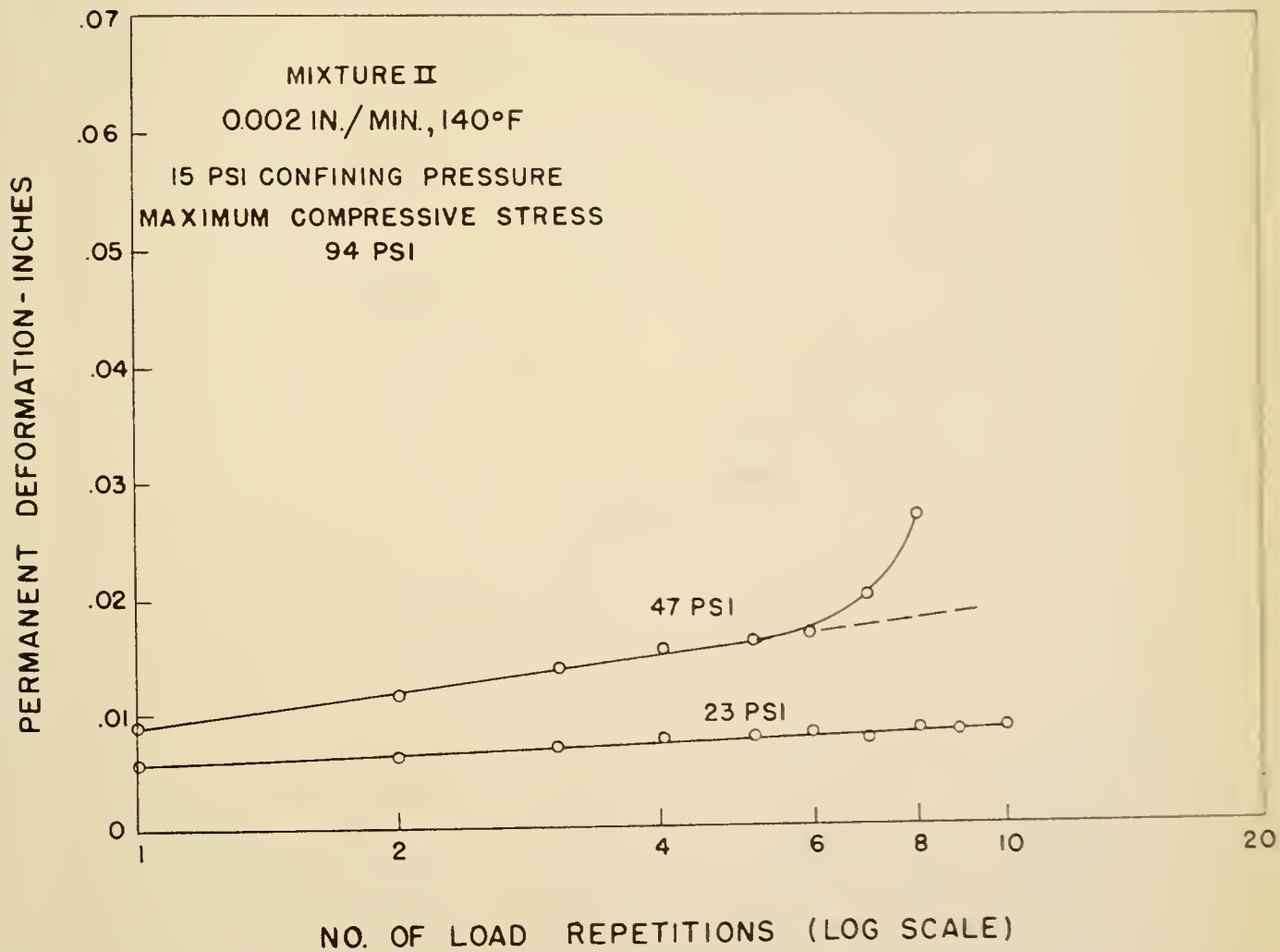
FIGURE 39

RELATIONSHIP   BETWEEN   PERMANENT   DEFORMATION

AND   NUMBER   OF   LOAD   REPETITIONS

FOR   VARIOUS   APPLIED   STRESSES

NO. OF LOAD REPETITIONS (LOG SCALE)
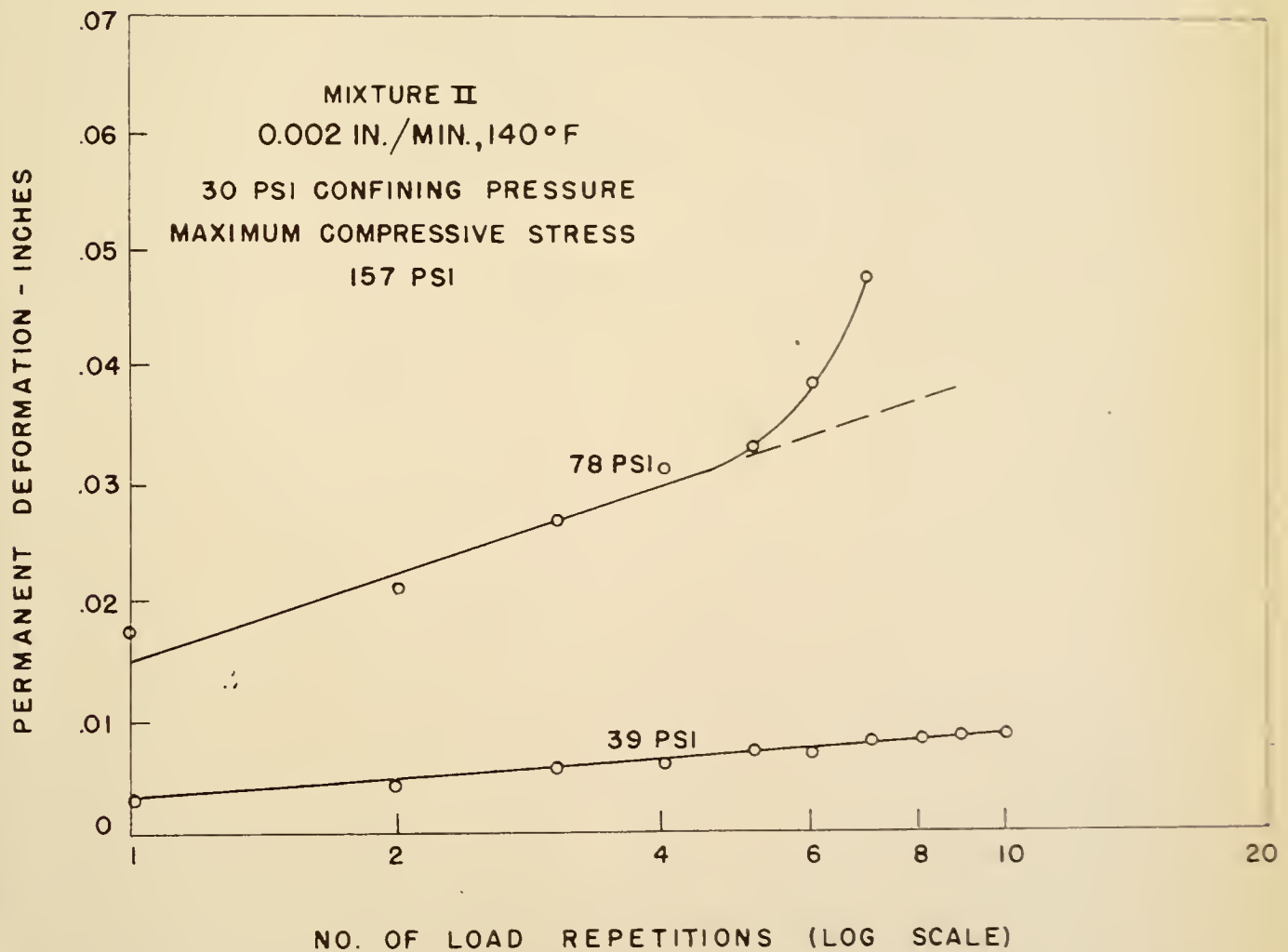
FIGURE 40

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES

MIXTURE II
0.02 IN./MIN., 100°F
15 PSI CONFINING PRESSURE
MAXIMUM COMPRESSIVE STRESS
114 PSI

85 PSI

57 PSI

28 PSI

PERMANENT DEFORMATION – INCHES

NO. OF LOAD REPETITIONS (LOG SCALE)
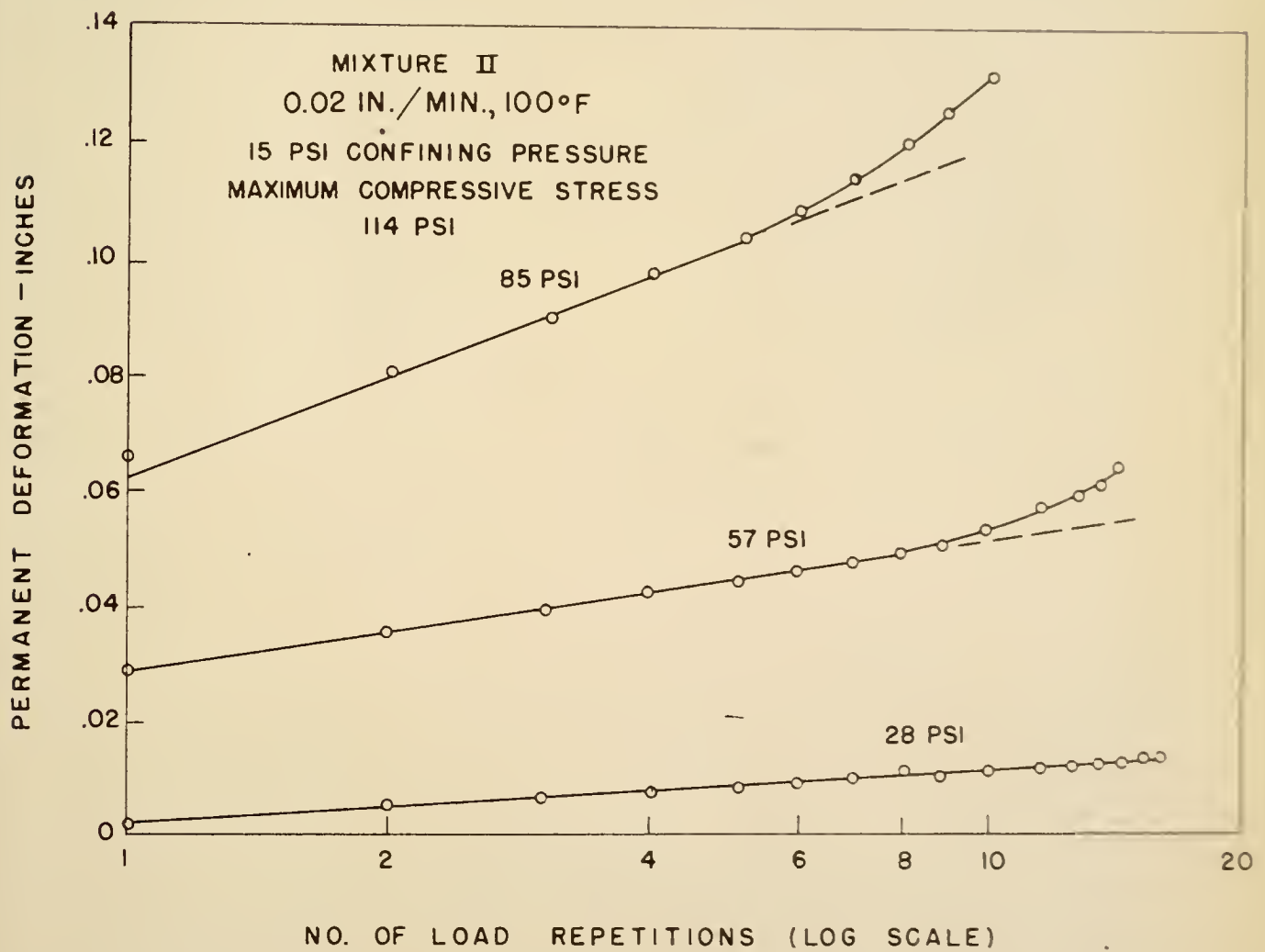
FIGURE 41

RELATIONSHIP BETWEEN PERMANENT DEFORMATION

AND NUMBER OF LOAD REPETITIONS

FOR VARIOUS APPLIED STRESSES
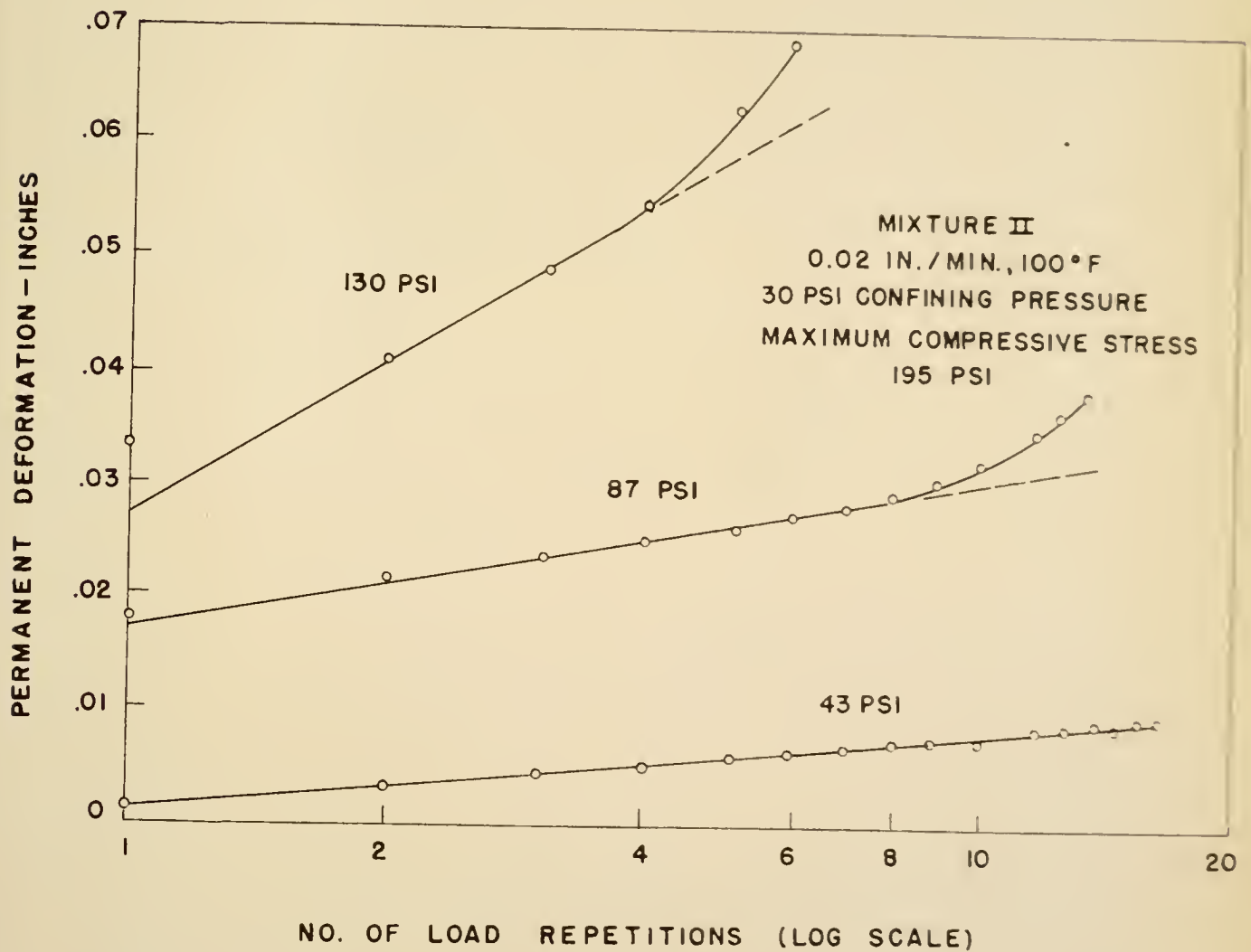
FIGURE 42

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITION
FOR VARIOUS APPLIED STRESSES

MIXTURE II
0.2 IN./MIN., 40°F

15 PSI CONFINING PRESSURE
MAXIMUM COMPRESSIVE STRESS
1080 PSI

540 PSI

270 PSI

PERMANENT DEFORMATION - INCHES

NO. OF LOAD REPETITIONS (LOG SCALE)
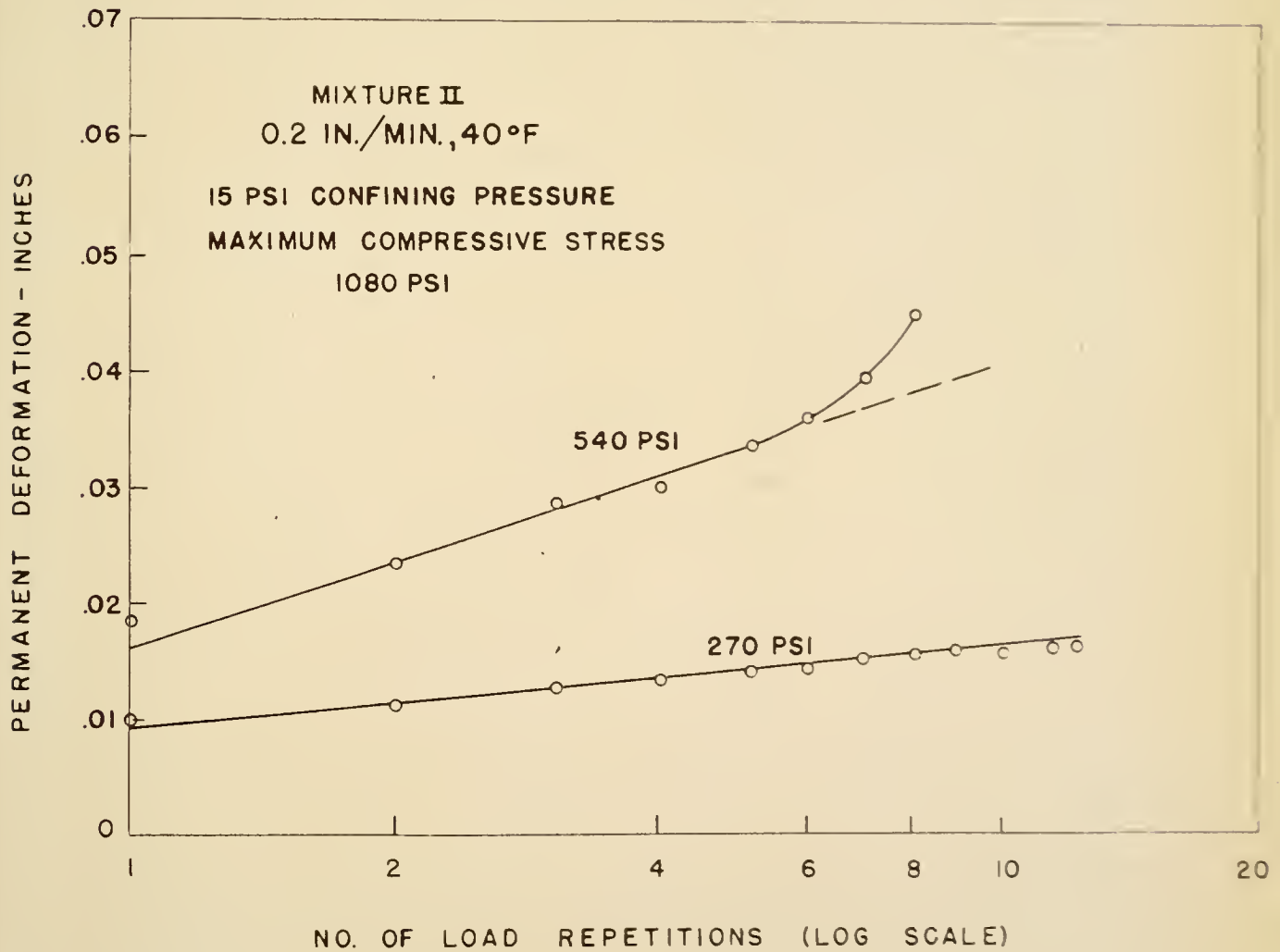
FIGURE 43

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES

MIXTURE II
0.2 IN./MIN., 40°F

30 PSI CONFINING PRESSURE
MAXIMUM COMPRESSIVE STRESS
1100 PSI

550 PSI

275 PSI

PERMANENT DEFORMATION – INCHES
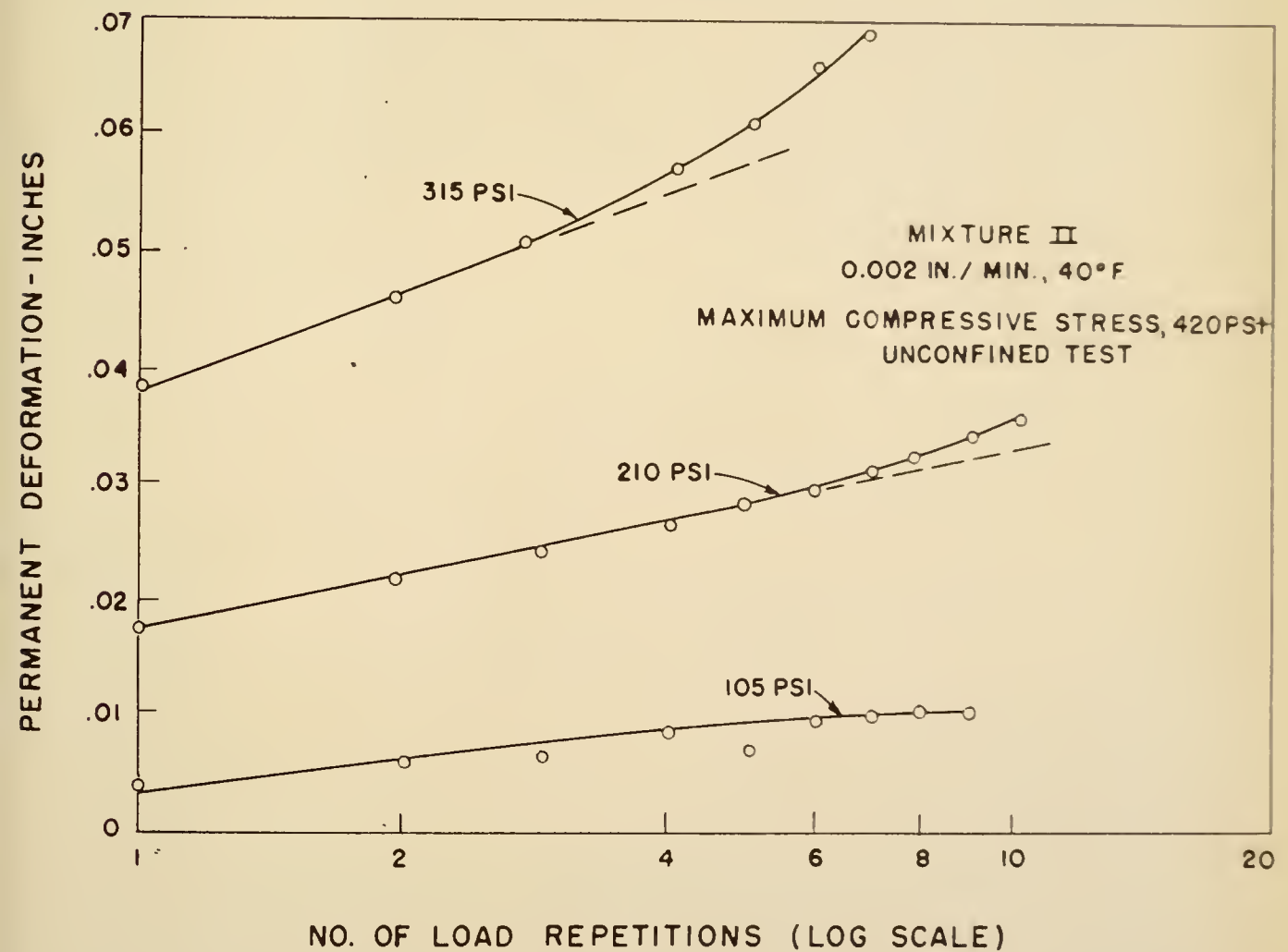
NO. OF LOAD REPETITIONS (LOG SCALE)
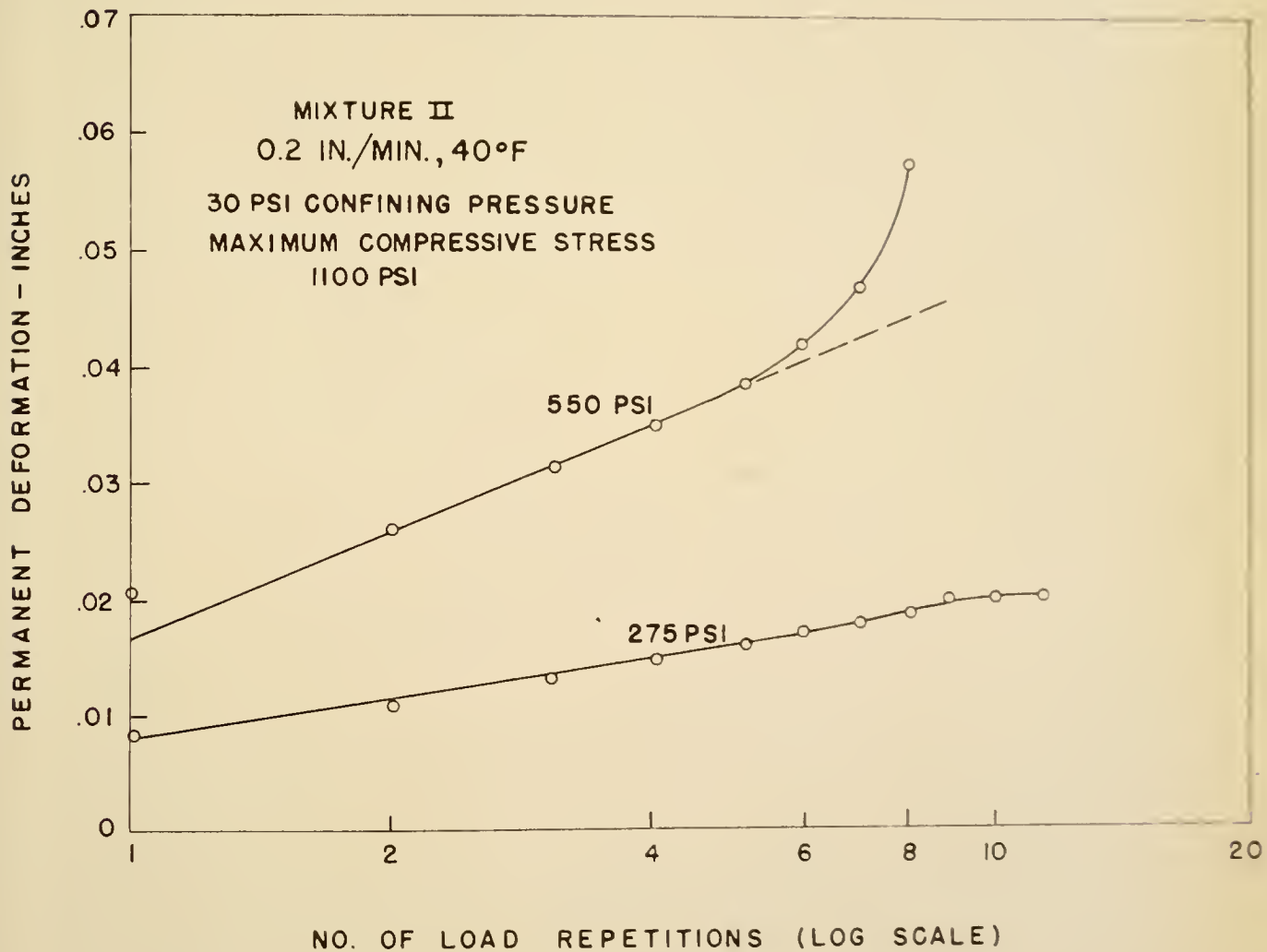
FIGURE 44

RELATIONSHIP BETWEEN PERMANENT DEFORMATION
AND NUMBER OF LOAD REPETITIONS
FOR VARIOUS APPLIED STRESSES
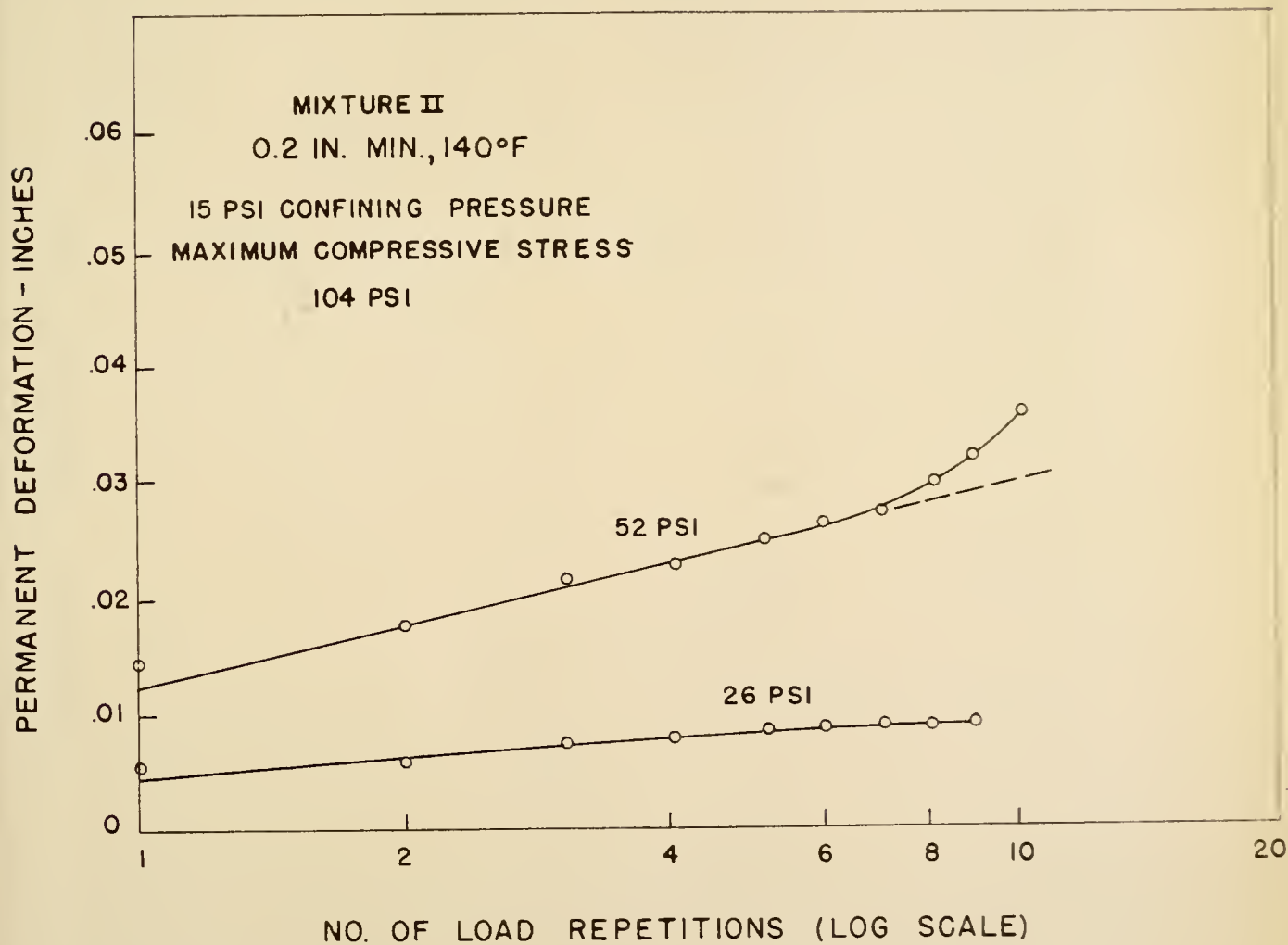
FIGURE 45

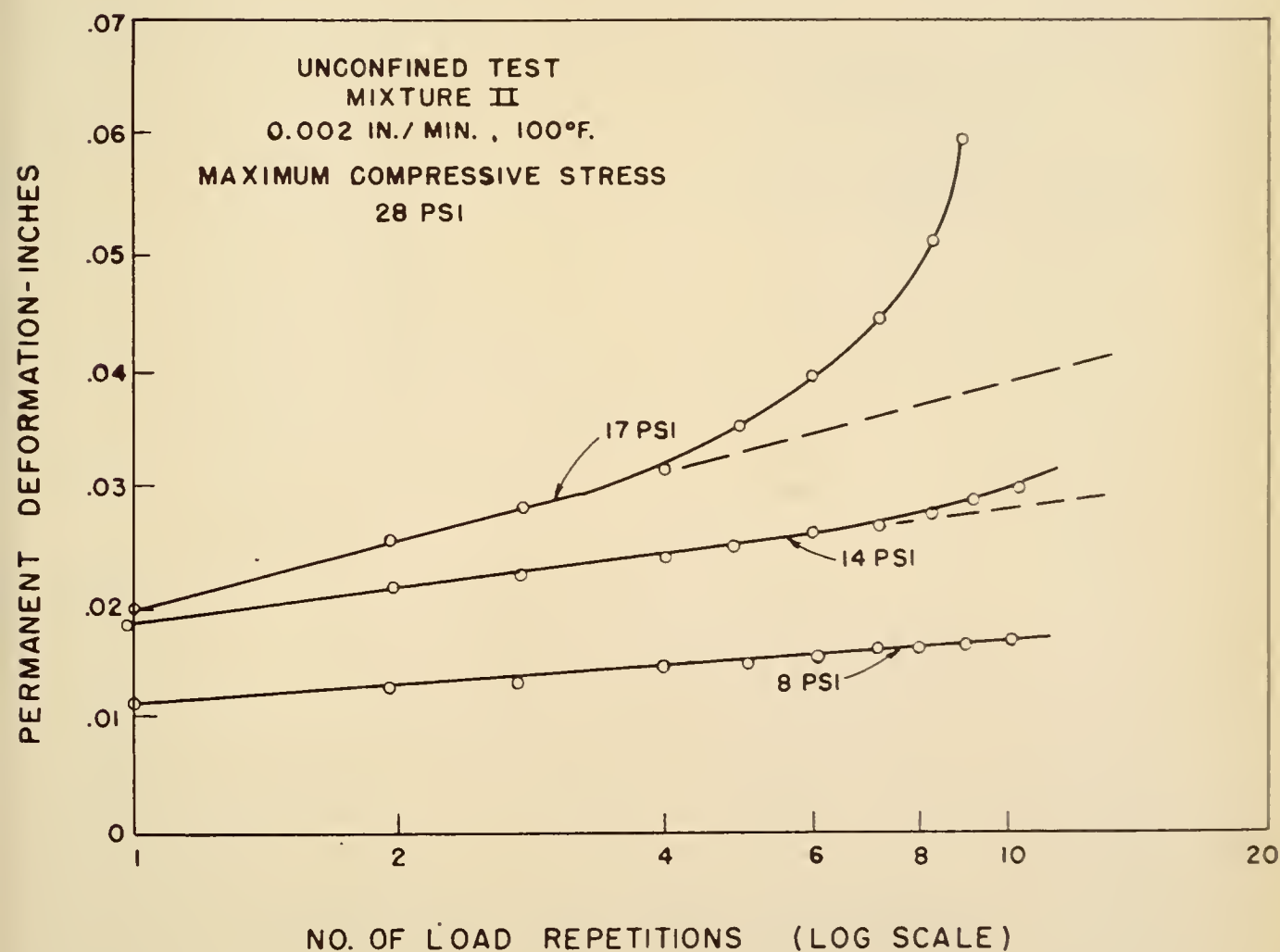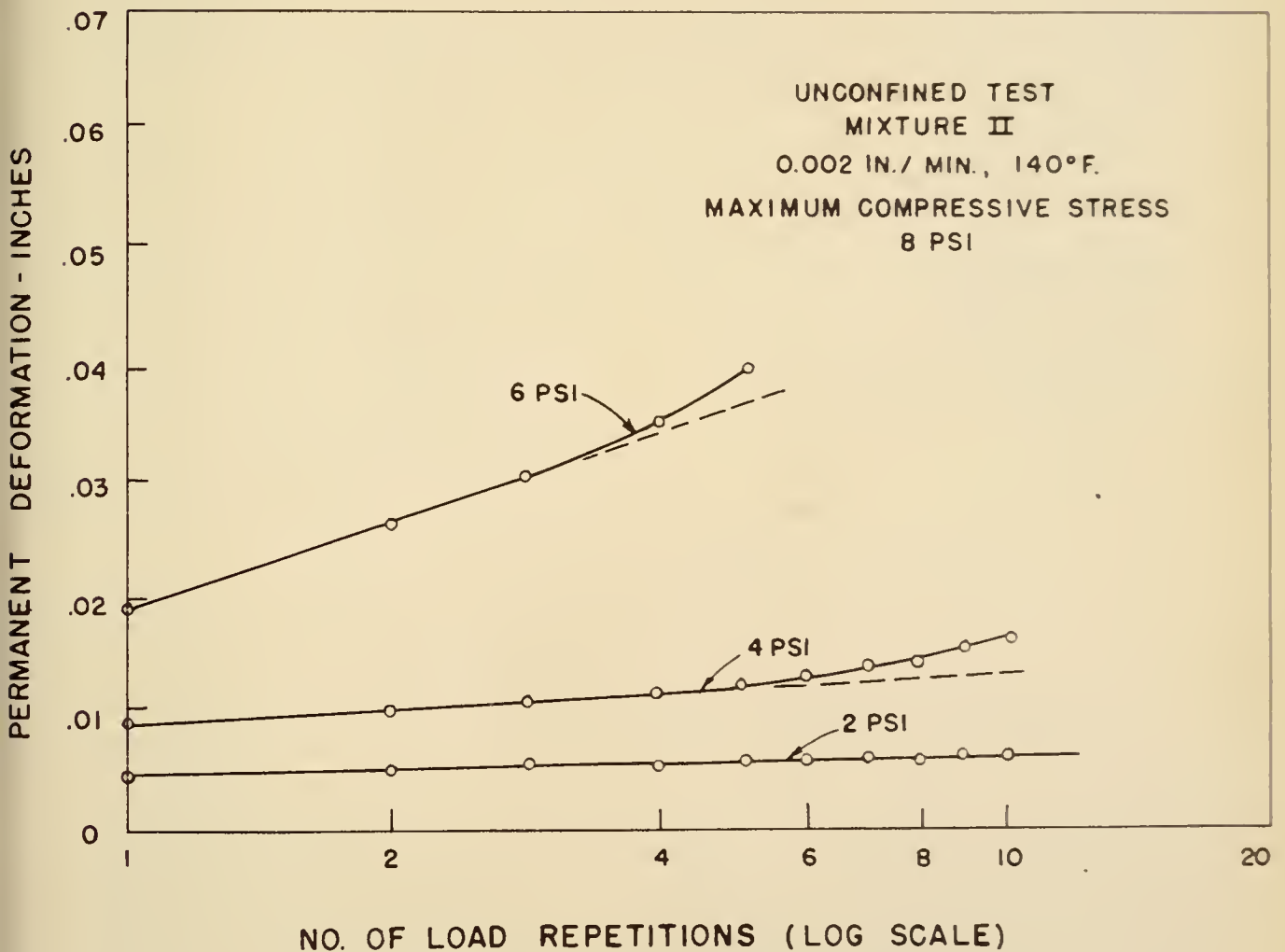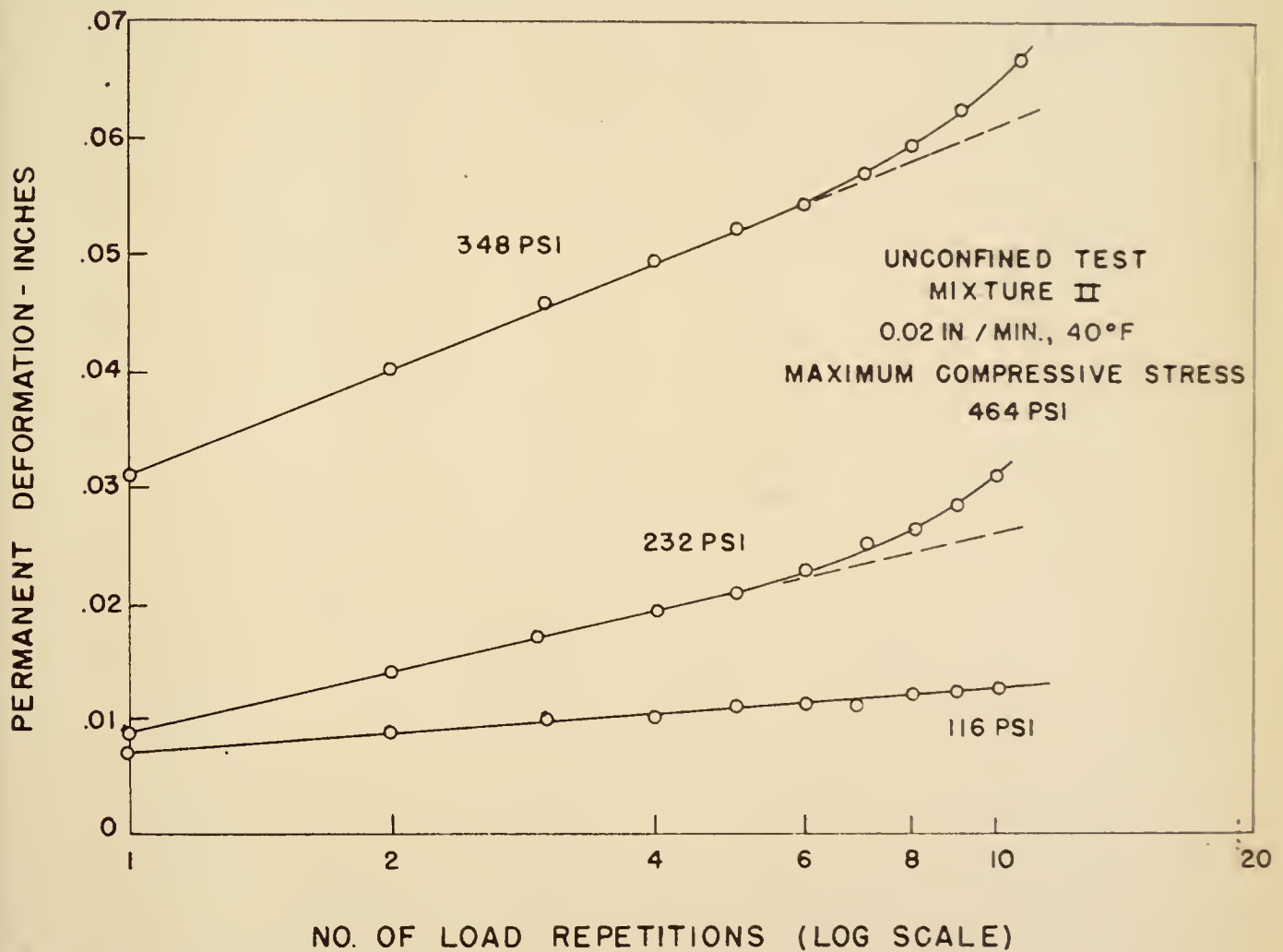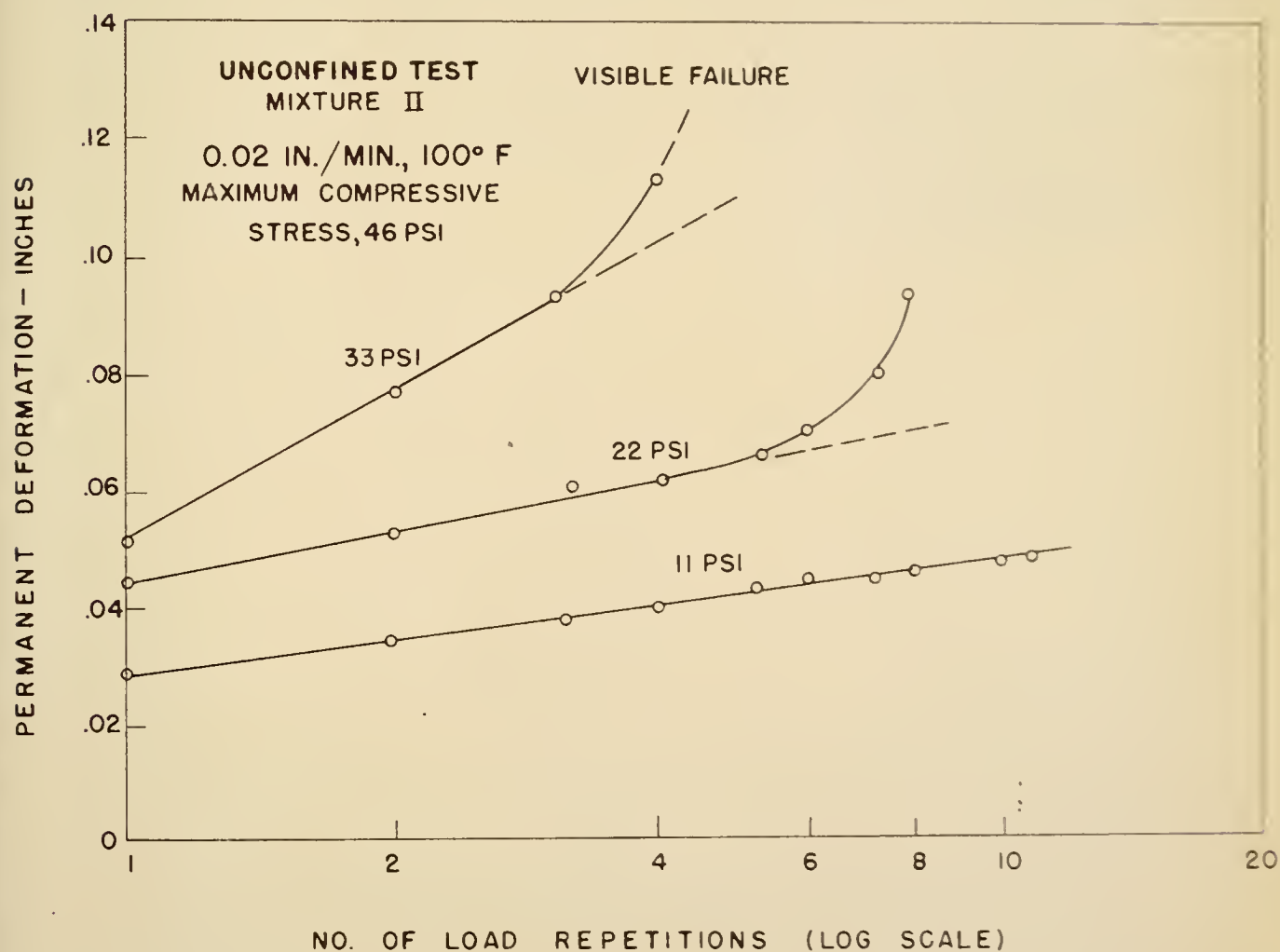RELATIONSHIP BETWEEN PERMANENT DEFORMATION AND NUMBER OF LOAD REPETITIONS FOR VARIOUS APPLIED STRESSES
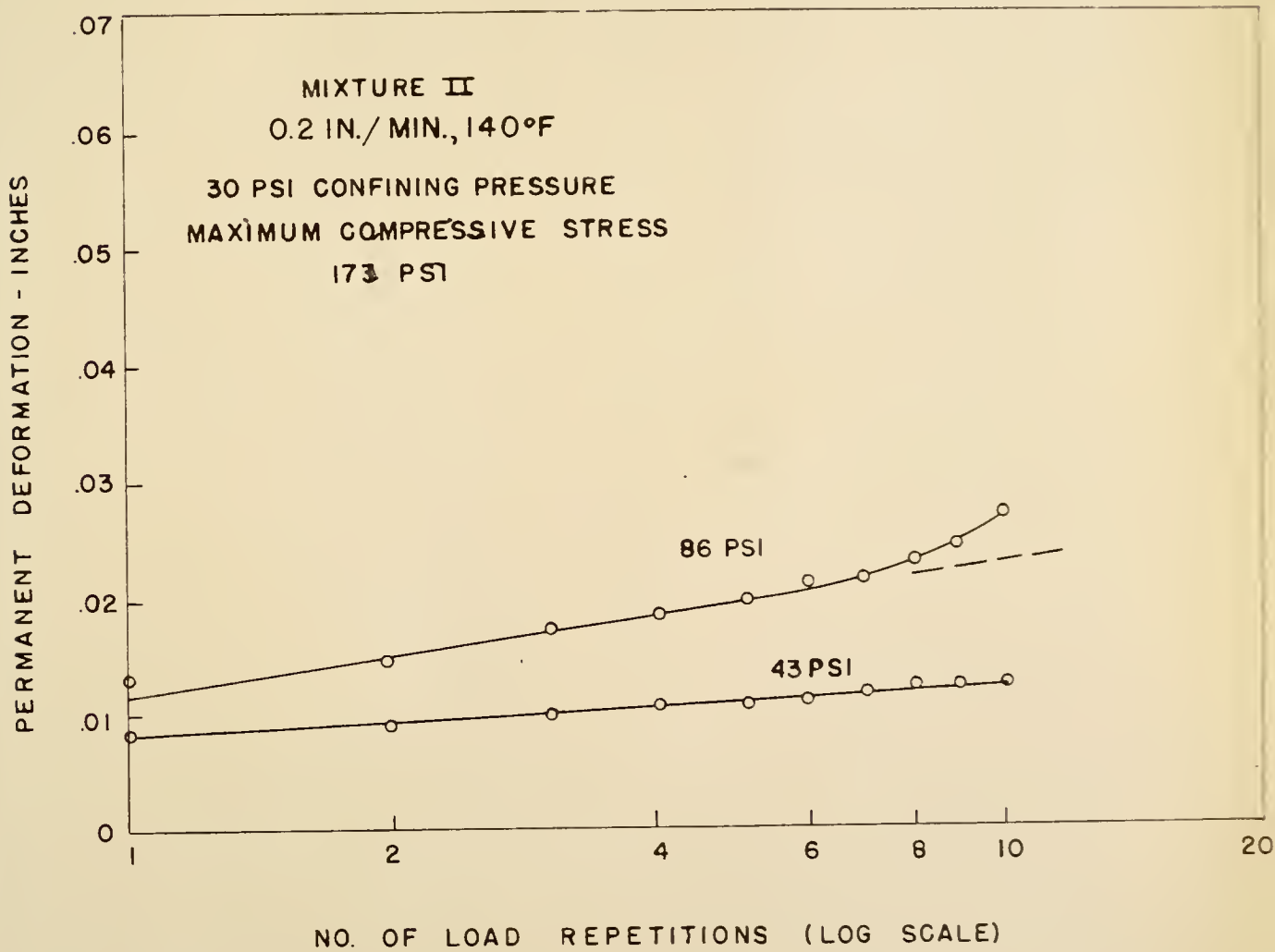
FIGURE 46

Table 10

Confined Repeated Load Test Results for Mixture II

Temperature °F	Lateral Pressure - psi	% Max. Comp. Stress	Rate of Deformation - in./min.		
			0.002	0.02	0.2
		Load Repetitions Necessary to Cause Failure.[1]			
40	15	50	5		5
	15	25	**		**
	30	50	7		5
	30	25	**		**
100	15	75		5	
	15	50		9	
	15	25		**	
	30	75		4	
	30	50		8	
	30	25		**	
140	15	50	6		7
	15	25	**		**
	30	50	5		6
	30	25	**		**

[1] For definition of failure, see text.

** Number of load repetitions used during test did not cause failure.

APPLIED STRESS ~ ENDURANCE LIMIT
RELATIONSHIPS AT VARIOUS TEMPERATURES
AND RATES OF LOADING

MIXTURE II
CONFINED TEST
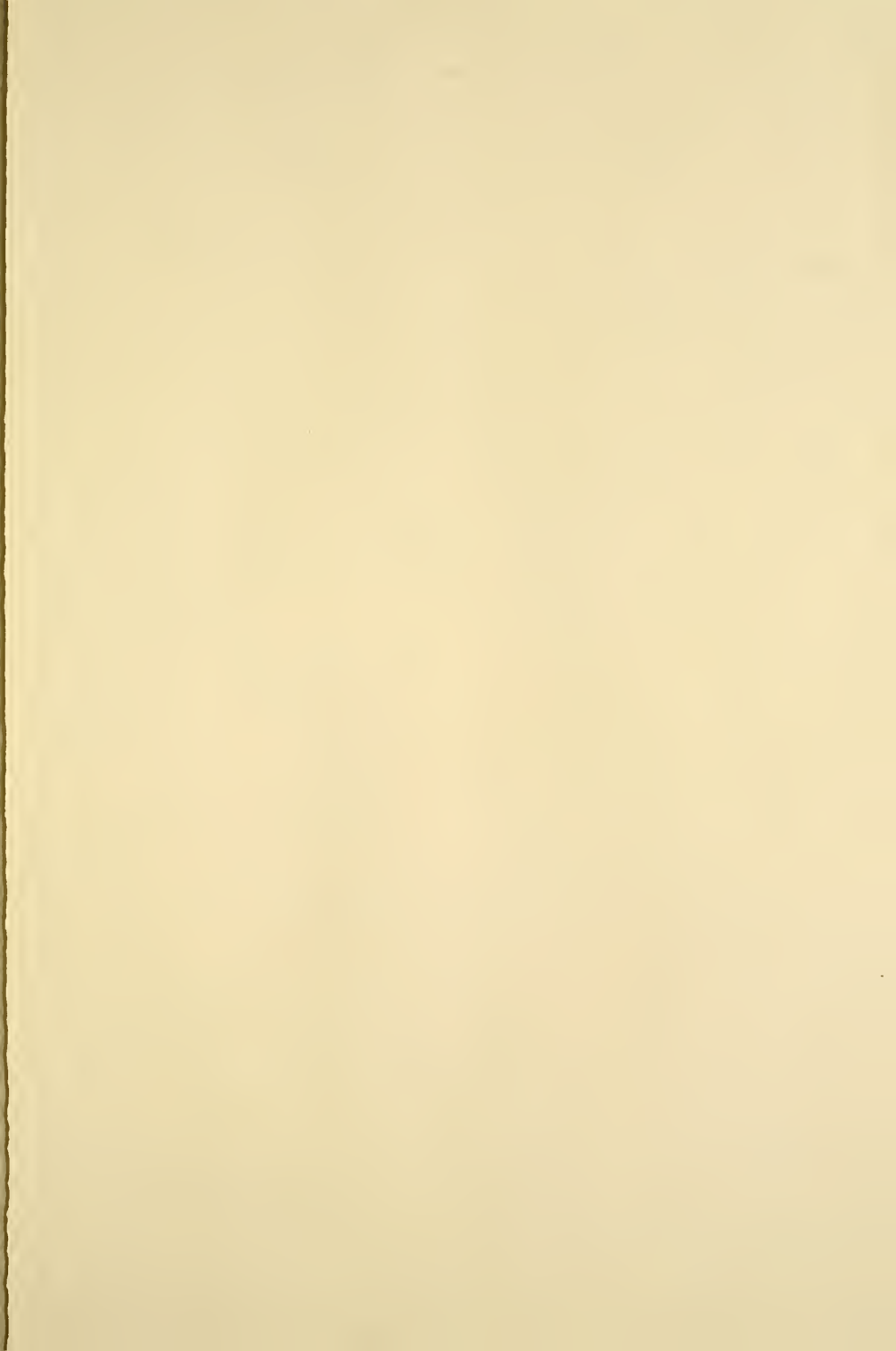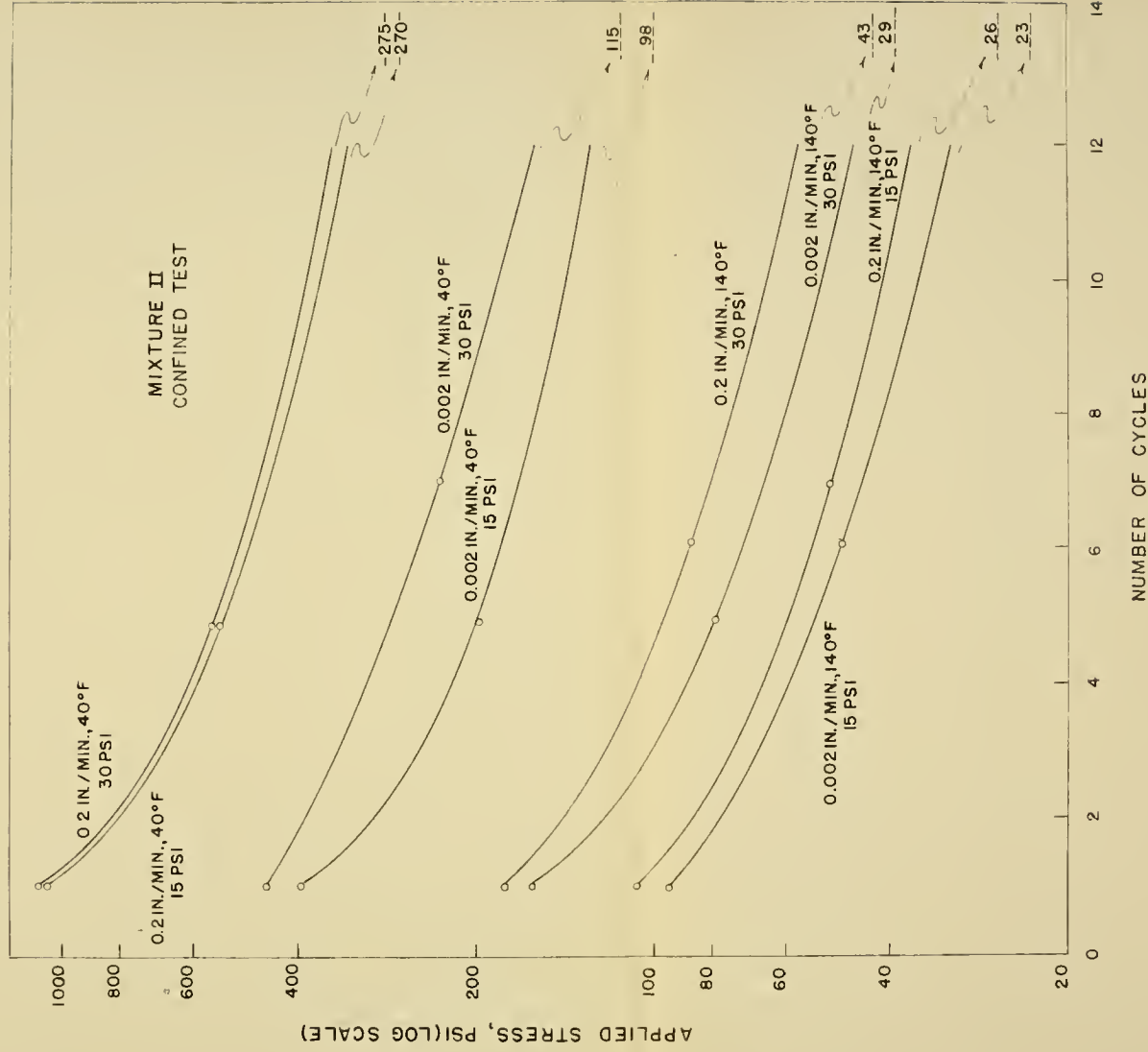
FIGURE 47

Static Unconfined Load Tests

The static unconfined load test series was performed to evaluate
the ability of a specimen to withstand a static load at five tempera-
tures: $40°$, $55°$, $70°$, $100°$, and $140°F$. The results of this study are
shown graphically in Figures 48 through 52 where the deformation is
plotted versus the time for various applied stresses at one temperature.

The maximum static stress the specimen withstood (as explained in
the Apparatus and Procedure section) at various temperatures is pre-
sented in Table 11 and graphically depicted in Figure 53 where the maxi-
mum unconfined static stress is plotted against the temperature.

RELATIONSHIP BETWEEN DEFORMATION AND
TIME FOR VARIOUS STATIC LOADS

MIXTURE II
140° F
UNCONFINED TEST

REBOUND

20 PSI

15 PSI

10 PSI

MAXIMUM STATIC STRESS

TIME – SECONDS (LOG SCALE)

FIGURE 48

DEFORMATION – INCHES
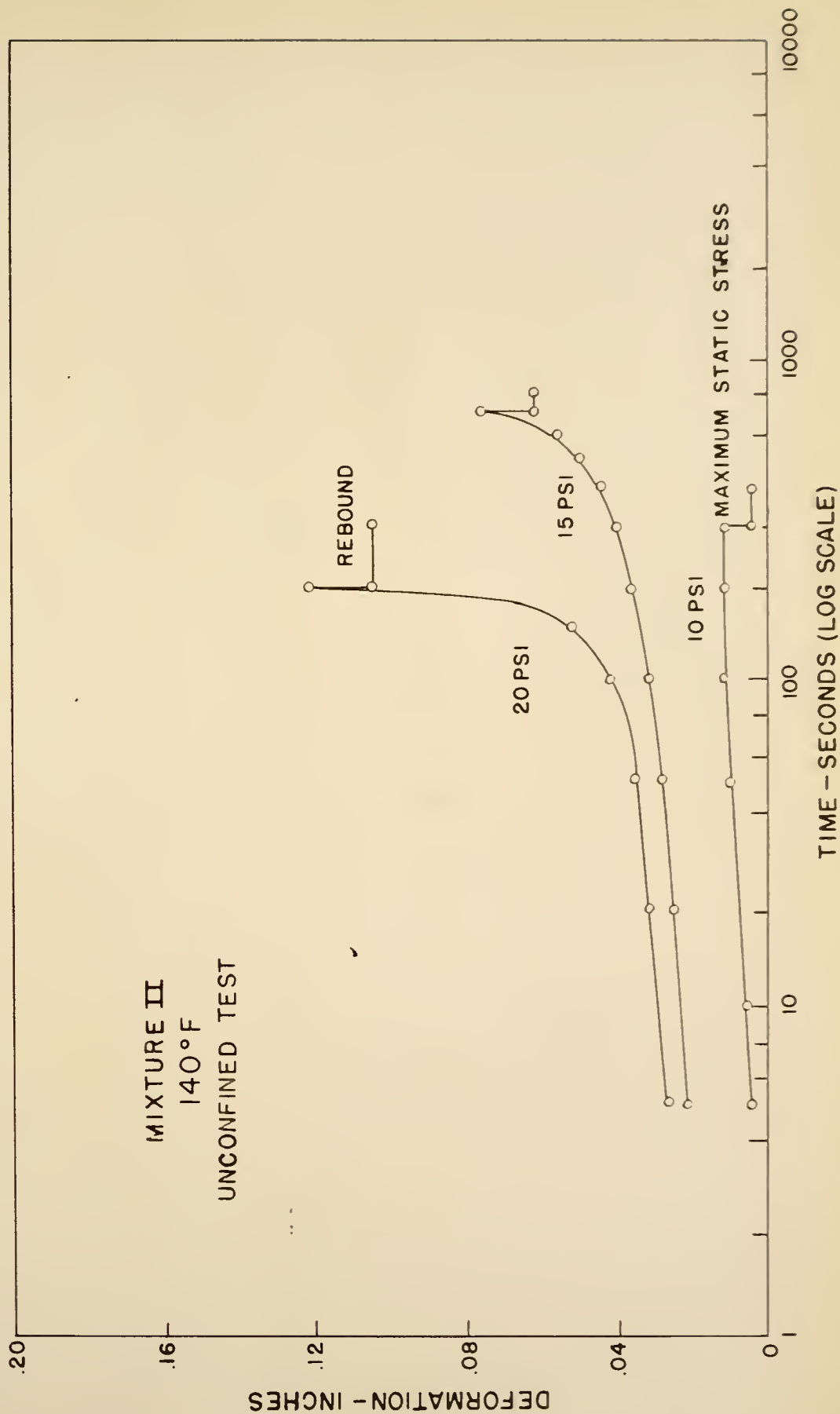
RELATIONSHIP BETWEEN DEFORMATION AND
TIME FOR VARIOUS STATIC LOADS
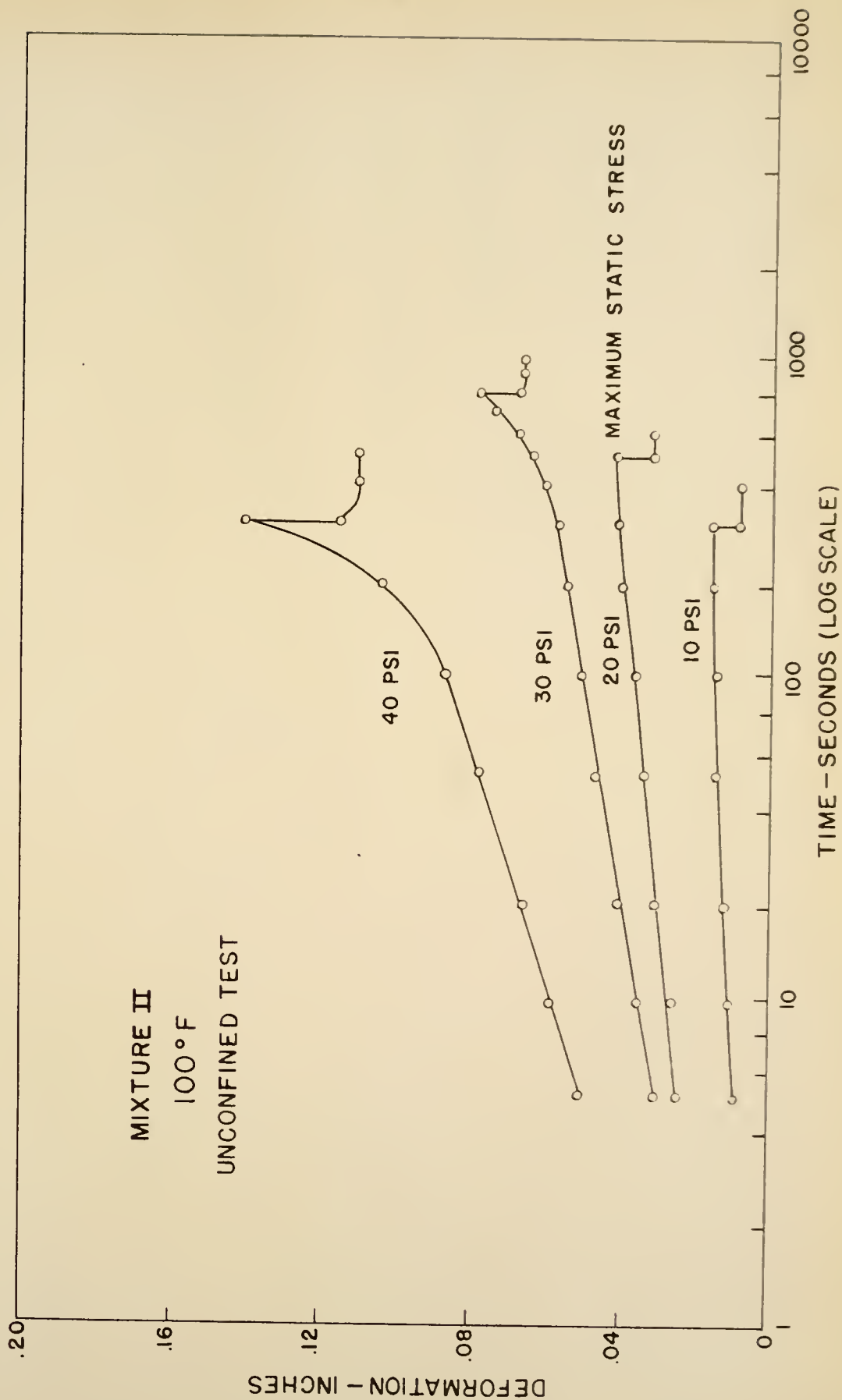
FIGURE 49

RELATIONSHIP BETWEEN DEFORMATION AND
TIME FOR VARIOUS STATIC LOADS

MIXTURE II
70°F
UNCONFINED TEST

110 PSI

MAXIMUM STATIC
STRESS

100 PSI

75 PSI

TIME—SECONDS (LOG SCALE)

DEFORMATION—INCHES
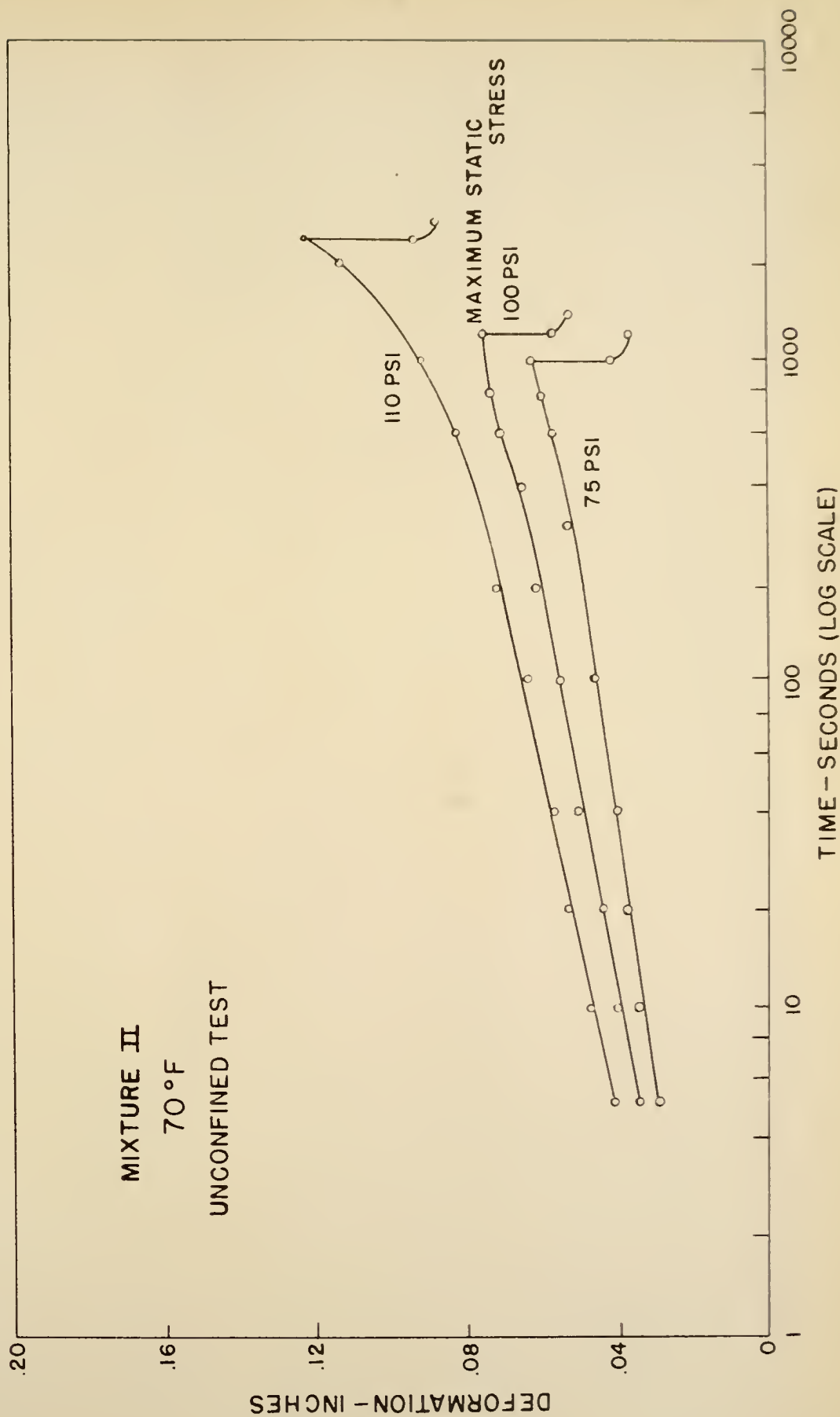
FIGURE 50

RELATIONSHIP BETWEEN DEFORMATION AND
TIME FOR VARIOUS STATIC LOADS

MIXTURE II
55°F
UNCONFINED TEST

MAXIMUM STATIC
STRESS

225 PSI

210 PSI

DEFORMATION—INCHES

TIME—SECONDS (LOG SCALE)
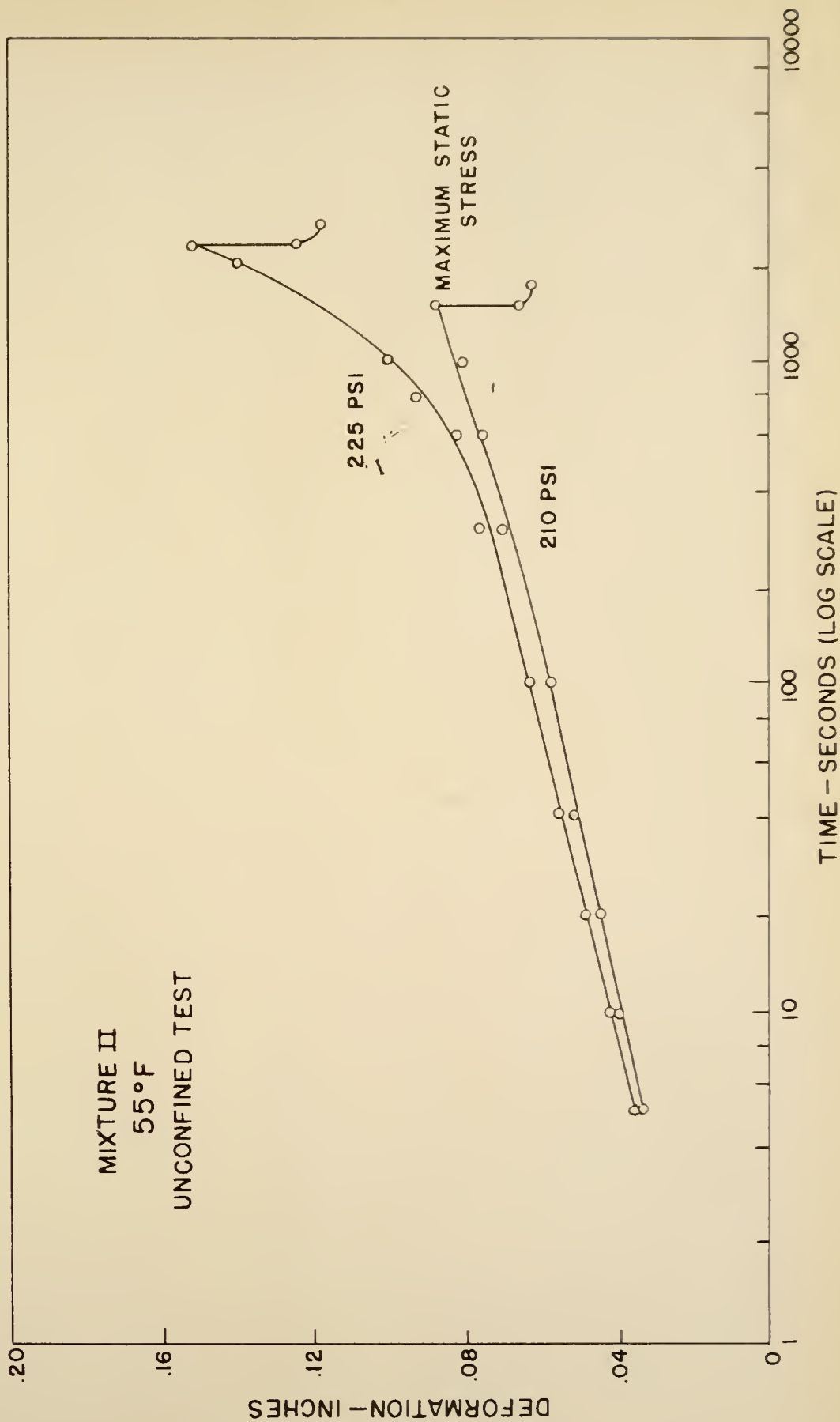
FIGURE 51

RELATIONSHIP BETWEEN DEFORMATION AND
TIME FOR VARIOUS STATIC LOADS
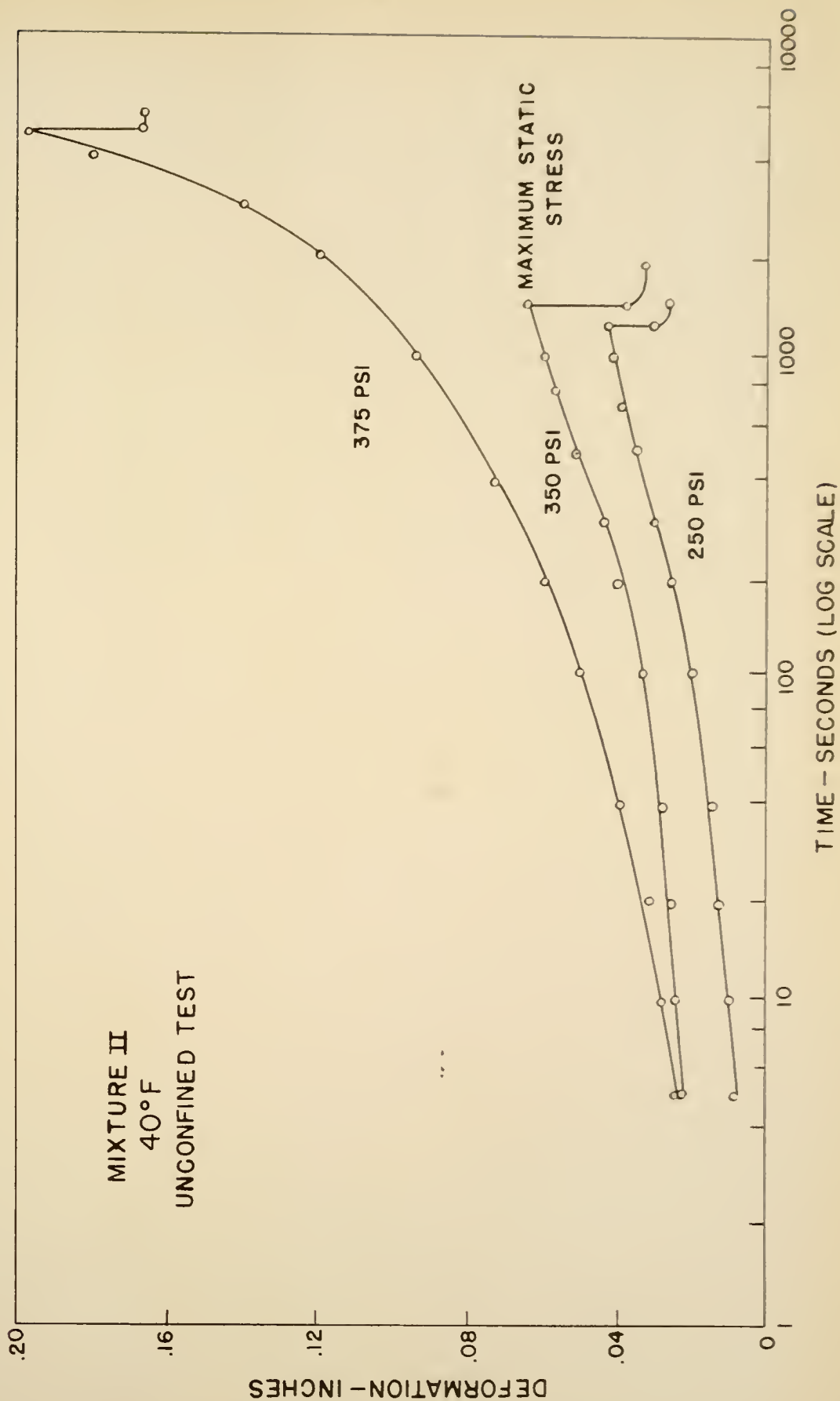
FIGURE 52

Table 11

Static Load Test Results for Mixture II

Temperature	Ultimate Static Stress - psi
40	350
55	200
70	100
100	20
140	10

RELATIONSHIP BETWEEN MAXIMUM STATIC
STRESS AND TEMPERATURE

MIXTURE II
UNCONFINED TEST
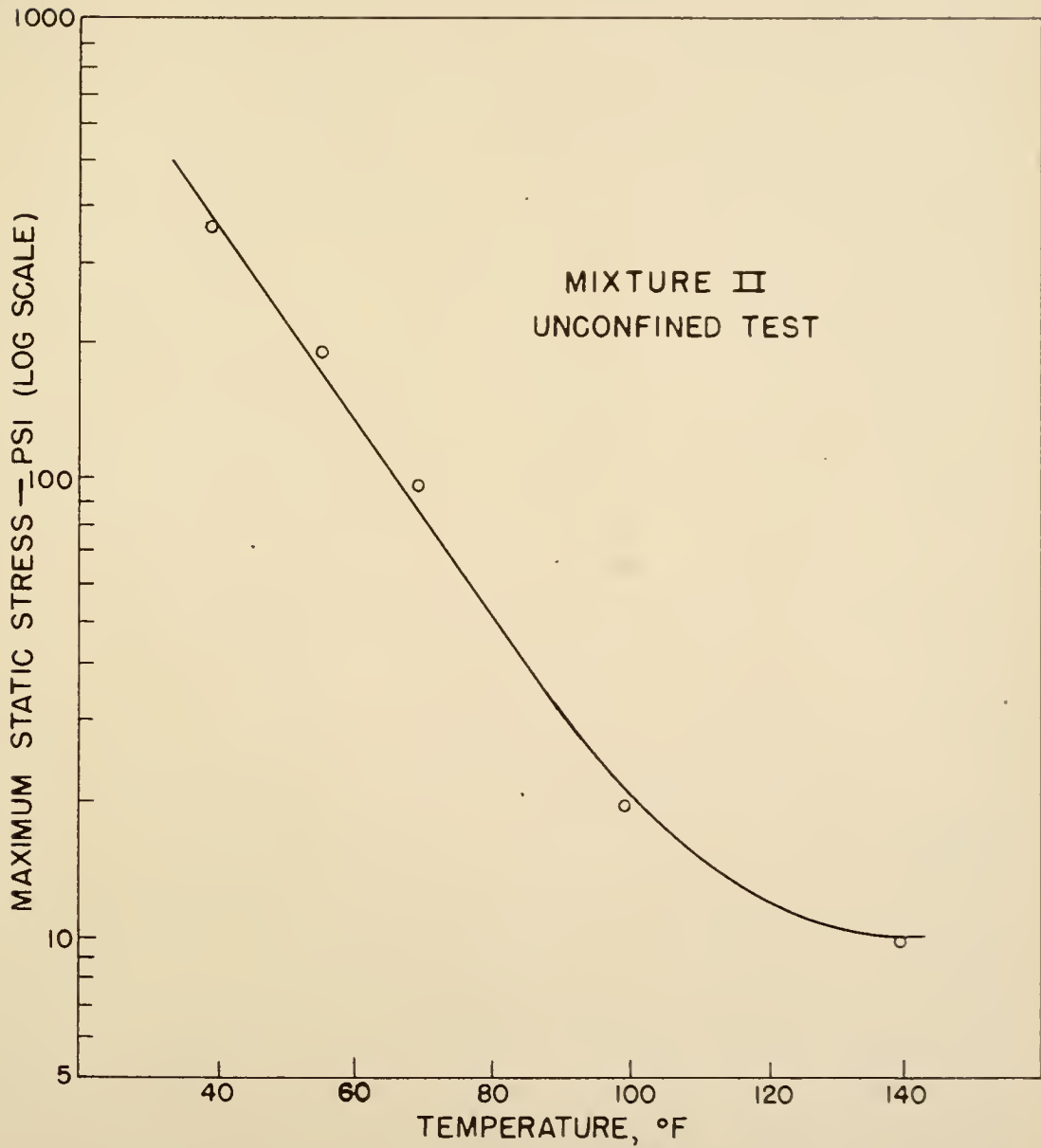
FIGURE 53

REGRESSION ANALYSIS AND STATISTICAL INFERENCES

Many of the data of this investigation were analyzed by a standard statistical procedure known as regression analysis (54). This was done in order to obtain prediction equations relating maximum compressive stress (confined and unconfined), temperature, rate of deformation, and number of load repetitions for the various mixtures tested.

To utilize an analysis of this type there are three assumptions which should be met. "These are: 1. that the observations are representative of the situation to which they are taken as referring; 2. that the observations are independent of one another; 3. that the deviations from the regression line are normally distributed with the same variability."(55)

The first two assumptions are generally met in most investigations and should cause no trouble. The third assumption implies that variation in the results for which we cannot account is caused by a number of independent events each of which contributes a part of the observed deviation. The normality assumption further implies that large positive deviations occur as often as large negative deviations and that more small deviations than large deviations occur.

In order to determine what mathematical model will best fit the results, plots must be made which relate the variables in different forms. The variables quite often turn out to have a curvilinear association and for this case evaluation of the parameters becomes a difficult

task. A simplification can be accomplished by working a linear transformation. An example of this is as follows: $Y = aB^x$; take the logarithm to base 10. Rewrite the equation as: $\log Y = \log a + (\log B)x$, and let $\log Y = Z$, $\log a = A$, $\log B = C$, and $x = D$. Proper substitution gives $Z = A + CD$. This transformation lends itself to a simple least squares analysis on the linear equation. A least squares analysis develops when one evaluates the parameters A and C such that the sum of the squares of deviations about the prediction curve is a minimum. (For a good discussion on least squares analysis, see Reference 54, page 119.)

This type of analysis was performed on results from the following tests:

1. Unconfined Compressive Strength Test

    (a) Mixture I

    (b) Mixture II

    (c) Indiana AH type B Surface Course Mixture

2. Unconfined, Repeated Load Test

    Mixture II

3. Confined, Repeated Load Test

    Mixture II

The summary of these calculations is shown in Appendix B.

The regression equation can take three forms: 1) raw data, 2) normal data, and 3) standard data. The two regression equations which are of interest in this study are the raw data form and the standard data form. The standard data form is of interest since its parameters give the relative magnitude of importance of each variable used in the prediction

equation. The raw data form is of most interest since it is from this equation that one would normally make predictions of the maximum compressive stress by directly substituting the applicable values of temperature and rate of deformation.

Regression Analyses on Compressive Strength Test Results

The regression analyses for the compressive strength test results of the various mixtures and of the different types of tests are presented in this section as follows:

    I. Unconfined Compression Test, Mixture I

    II. Unconfined Compression Test, Mixture II

    III. Unconfined Compression Test, Indiana AH type B Surface Mixture

    IV. Confined Compression Test, Mixture II

Unconfined Compression Test, Mixture I

It was first desired to express the family of curves in Figures 19 and 20 in a prediction equation. The general mathematical form appeared to be:

$$x_0 = A^{Bx_1^{(Cx_2 + D)}} \tag{1}$$

By taking the log of the equation twice and grouping some constants one obtains:

$$\log \log x_0 = A' + B' \log x_1 + C'x_2 + D'x_2 \log x_1 \tag{2}$$

    where x_0 = maximum unconfined compressive stress, psi

          x_1 = rate of deformation, in./min.

          x_2 = temperature, $^{\circ}F$

          A, B, C, D, A', B', C', D' = constants.

Now letting $X_0 = \log \log x_0$

$$X_1 = B' \log x_1$$

$$X_2 = C' x_2$$

$$X_3 = D' x_2 \log x_1$$

and substituting these values in equation 2, one gets:

$$X_0 = C_0 + C_1 X_1 + C_2 X_2 + C_3 X_3 \qquad (3)$$

where C_0, C_1, C_2, and C_3 are constants.

The problem then becomes to evaluate C_0, C_1, C_2, C_3 such that one gets the minimum sum of squares of the deviations as discussed previously.

To simplify calculations, the raw data were coded and transformed so that:

$$X_0 = 10,000 \log \log x_0$$

$$X_1 = 1,000 \log 1,000 x_1$$

$$X_2 = x_2/20$$

$$X_3 = 50 x_2 \log 1,000 x_1.$$

Upon completion of the analysis, the raw data equation (coded and transformed) was:

$$X_0 = 5876.33 - 842.69X_2 + 0.0224X_1 + 0.1195X_3, \qquad (4)$$

which is the multiple regression equation.

(R^2) is defined as "the proportion of the sum of squares of the dependent variable which is explained by the multiple regression equation." (54) (R^2) computed for the laboratory data used in the above model was 0.997. This means that the above regression equation is a very good representation of the underlying relationship between the variables studied. Using this equation, an excellent estimation of the

unconfined, maximum compressive stress for many rates of deformation and temperatures could be determined for Mixture I.

It must be remembered that the regression equation estimates the average value of x_0, the maximum unconfined compressive stress. For example, one would wish to know what average unconfined compressive strength a series of bituminous mixture specimens would have when tested at a fixed set of conditions. By use of statistical procedures one may determine an interval within which he is quite certain the mean value will fall. One might also be interested in how much variation he should expect in individual observations. Individual observations are known to vary more widely than sample means. This follows from the third assumption which was discussed in the beginning of this section. The prediction interval for individual observations will necessarily be larger than the interval for the means. For a more thorough discussion on prediction intervals the reader is referred to any standard statistics text of this subject.

The multiple correlation coefficient (R) measures the degree of linear association among all of the variables (X_1, X_2, X_3) with X_0. (R) for the regression equation given was 0.998.

Another term used is the partial correlation coefficient (r) which "measures the degree of linear association between two variables after eliminating any linear tendency of other variables to affect jointly the two variables under consideration." (54)

For Mixture I the following partial correlation coefficients were obtained:

$$r_{01} = 0.5297, \quad r_{02} = -0.8182, \quad r_{03} = 0.0114.$$

r_{01} refers to the correlation between X_0 and X_1.

r_{02} refers to the correlation between X_0 and X_2.

In this case it can be seen that that correlation between the maximum compressive stress term and the temperature is much higher than the correlation between the maximum compressive stress term and the rate of deformation. The interaction term (X_3) is rather weakly correlated with the maximum unconfined compressive stress and yet it adds considerably to the strength of the relationship. As was previously mentioned, the square of the multiple correlation coefficient $(R^2_{0.123})$ equals 0.997. By dropping the interaction term (X_3) from the analysis, one obtains an $R^2_{0.12}$ equal to 0.95. This is a noticeable drop in the degree of association.

One should be aware of the danger involved in extrapolating beyond the range of observations. Unless one is reasonably certain that the determined function does exist over a wider range of values for the variables than in the present study, the regression equation should be used only within the limits of its determination. While excellent correlation was indicated over the range of temperature and rate of deformation used in this series, a wider range of values for those variables used in this series might indicate that another type of function would be necessary.

With the excellent correlation established using the given general mathematical model $x_0 = A^{Bx_1^{(Cx_2 + D)}}$, it should be possible with a limited number of tests to establish the relationship among maximum compressive stress, temperature, and rate of deformation for any chosen mix.

Unconfined Compression Test, Mixture II

To check the hypothesis that the parameters of the general mathe-
matical model expressed above could be determined for any chosen mix
with a limited number of tests, Mixture II was formulated as previously
mentioned and tested at the extreme values of temperature (40°F and 140°F)
and rate of deformation (0.002 in./min. and 0.2 in./min.). The results
from these test conditions were used to evaluate the parameters of the
regression equation for Mixture II (see equation 2). The analysis is
shown in Appendix B. Once the regression equation was established, it
was used to predict maximum unconfined compressive stresses at other
test conditions (100°F at 0.002, 0.02, and 0.2 in./min. and 0.02 in./min.
at 40°F and 140°F). Specimens were then tested under these test condi-
tions. The predicted values and the observed values are both shown in
Figure 21. Very good correlation between the predicted values and the
observed values were obtained in all cases but one: 40°F and 0.02 in./min.
No reason is advanced for this discrepancy.

Unconfined Compression Test, Indiana AH Type B Surface Mixture

The original model (equation 1) was derived for results obtained
for a sand-asphalt mixture tested in the unconfined compression test.
This mixture was quite plastic in character. It was felt that the orig-
inal model should be applicable to a wide range of mixtures. To test
this hypothesis, specimens were formulated from a crushed-stone aggre-
gate (1/2 inch maximum size) following the Indiana AH type B surface
course gradation. This mixture was less plastic in character, had a

lower asphalt content, and had aggregate particles that were much more angular in shape than the sand-asphalt mixture.

The specimens formulated from the Indiana AH type B surface course mixture were tested to failure in the unconfined state at the extreme conditions of temperature (40°F and 140°F) and rate of deformation (0.002 in./min. and 0.2 in./min.). The results from tests at these conditions were used to establish the parameters of the regression equation. The analysis is shown in Appendix B. The established regression equation was used to predict the maximum unconfined compressive stresses at the other test conditions (100°F at 0.002, 0.02, and 0.2 in./min. and 0.02 in./min. at 40°F and 140°F). Specimens were then tested at the above test conditions. The predicted values and the observed values are both shown in Figure 22. Very good correlations were observed at all levels between the observed values and the predicted values. This would indicate that the general expression is valid for the unconfined condition for both sand-asphalt and bituminous-concrete mixes.

Confined Compression Test, Mixture II

The original model (equation 1) was derived for results obtained from the unconfined compression test. It was hoped that the original model would have wider applicability than for just the unconfined test condition. Since mixtures are loaded in the field in such a manner that some degree of lateral support is provided, it was decided to check the validity of equation 1 by performing compression tests at various confining pressures.

Compression tests were made at two confining pressures: 15 and 30 psi. At each of the confining pressures and at the extreme values of

temperature (40°F and 140°F) and rate of deformation (0.002 in./min. and 0.2 in./min.) specimens from Mixture II were tested to failure. The results from tests at these test conditions again were used to evaluate the parameters of the regression equation. This analysis is shown in Appendix B. The regression equations, once established, were used to predict maximum compressive stresses at the other test conditions (100°F at 0.002, 0.02, and 0.2 in./min. and 0.02 in./min. at 40°F and 140°F). Specimens were then tested at the above test conditions at both 15 and 30 psi. Figure 23 shows both the predicted values and the observed values for a confining pressure of 15 psi. Good correlation between observed and predicted values was noted except in the case of tests made at 100°F temperature. For all three rates of deformation at 100°F the predicted values were higher than the observed values.

From observation of the results of tests on Mixture II, it can be seen that the introduction of lateral support increases the maximum confined compressive strength of the mixture more at the higher temperatures than at lower temperatures. It is at these higher temperatures that the mixture is quite plastic in character. At 40°F the mixture loses most of its plastic nature and it is relatively "stiff." Therefore, the introduction of lateral pressure at 40°F resulted only in a small increase in the maximum confined compressive strength of the mixture. The proportional change of the maximum confined compressive strength of the mixture with temperature at any one rate of deformation is less pronounced under conditions of confinement than under conditions where no confinement exists. With the inclusion of the results at 100°F into the regression

analysis a much better fitting regression equation could be found than was obtained using limited data from the extreme conditions of temperature and rate of deformation.

Figure 24 shows the predicted values and the observed values for a confining pressure of 30 psi. Again good correlation between the predicted and observed values was noted except in the case of the $100^{\circ}F$ temperature. For all three rates of deformation at $100^{\circ}F$, the predicted values were higher than the observed values. It is obvious that enough change in mixture strength resulted from the introduction of confining pressures that one would no longer be secure in evolving a prediction equation for wide ranges based upon limited results from four test levels. The addition of an interaction term involving the confining pressure and temperature to the original model might increase its effectiveness when it comes to the case of tests made under the confined condition.

Regression Analysis on Unconfined
Repeated Load Results for Mixture II

Since mixtures in the field are subjected to numerous loadings, it was decided to establish a test procedure that would duplicate this action of repeated loads. Figure 36 shows the results of a series of tests where specimens made from Mixture II were subjected in the unconfined condition to repeated loads whose magnitude was some percentage of the maximum unconfined compressive stress determined at a given rate of deformation and temperature. From Figure 36 came the concept of an endurance limit for a bituminous mixture tested under the above-listed conditions. The endurance limit is defined as the maximum stress to which a mixture may be subjected a large number of times without failure. (For a discussion of a failure criterion, see the section on Results.)

Figure 36 shows the effect of temperature and rate of deformation upon the endurance limit. Regression analysis was used to establish an equation for the family of curves shown in Figure 36. From the plot it was determined that a model for any one test condition was:

$$x_c = \left[E \cdot 10^{-\alpha(n-1)^\beta} + (1 - E) \right] x_0$$

where x_c = applied, cycled, unconfined compressive stress, psi

x_0 = maximum unconfined compressive stress for the test

condition (fixed rate of deformation and temperature)

n = number of load repetitions necessary to cause

excessive shear deformations

E, α, β = parameters that are dependent upon mixture composition.

From this model it can be seen that when n = 1, $x_c = x_0$. This is not exactly correct according to the failure criterion developed since this means the specimen could withstand one cycle of the maximum unconfined compressive stress. However, in the interest of a general expression, it was felt that the discrepancy was not a major source of error. It can be seen that as the magnitude of the applied stress is decreased, the number of loading cycles that a specimen will withstand before failure is greater. The limiting value for x_c, which was called the endurance limit, is approached as (n) gets large and the term $E \cdot 10^{-\alpha(n-1)}$ approaches 0. In this situation, the endurance limit equals $(1 - E)x_0$. For the mixture used in this series of tests $(1 - E)$ was 0.25.

The regression analysis used to establish the numerical values of the parameters is shown in Appendix B. The regression equation in final form is as follows:

$$x_c = \left[0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25\right] x_0 \qquad (6)$$

General Equation Relating Temperature, Rate of Deformation, Stress, and Repeated Loads

In this section the results of the study on the effect of temperature and rate of deformation upon the maximum unconfined compressive stress and the results of the study on the effect of repeated loads are combined into one general expression:

$$\log \log x_c = A + B \log x_1 + Cx_2 + Dx_2 \log x_1 \left[E \cdot 10^{-\alpha(n - 1)^\beta} + (1 - E)\right] \qquad (7)$$

where x_c = applied, cycled, unconfined compressive stress, psi

x_1 = rate of deformation, in./min.

x_2 = temperature, ${}^\circ F$

n = number of load repetitions necessary to cause failure

A, B, C, D, E, α, and β = constants that are dependent

upon mixture composition.

This model can be used to evaluate the effect of temperature, rate of deformation, and applied stress upon the number of load repetitions that an unconfined, rational-sized specimen will withstand before the failure criterion is reached. By setting $n = 1$, it is possible to determine the maximum unconfined compressive stress for numerous combinations of test conditions. Then $x_c = x_0$. The final equation for Mixture II with the numerical values is as follows:

$$\log \log x_0 = 0.587633 - 0.004213 \ x_2 + 0.00224 \ \log 1000 \ x_1 +$$

$$0.000597 \ x_2 \ \log 1000 \ x_1 \left[0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25\right] \qquad (8)$$

An example illustrating the use of this expression is shown as follows:

Example: Test Conditions: Temperature $60^{\circ}F$, Rate of Deformation 0.01 in./min.

(1) Determine the predicted, unconfined, maximum compressive strength of a sand-asphalt mixture at the given test conditions.

$$\log \log x_0 = 0.587633 - 0.004213x_2 + 0.00224 \log 1000 \, x_1 + 0.000597x_2$$
$$\log 1000 \, x_1 \, (0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + .25)$$

Now $n = 1$

so: $\log \log x_0 = 0.587633 - 0.004213(60) + 0.0022 \log 1000 \, (0.01) +$
   $0.000597(60) \log 1000 \, (0.01) \left[(0.75 + 0.25)\right]$

By collecting terms

$$\log \log x_0 = 0.372913$$
$$\log x_0 = 2.360$$
$$x_0 = 229 \text{ psi}$$

$\therefore$ the maximum unconfined compressive stress of the sand-asphalt mixture tested at $60^{\circ}F$ and 0.01 in./min. is 229 psi.

(2) Determine the number of load repetitions (n) necessary to reach the failure criterion in the unconfined state under an applied stress of 90 psi at a temperature of $60^{\circ}F$ and rate of deformation of 0.01 in./min.

$$x_c = x_0 \, (0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25)$$
$$90 = 229 \, (0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25)$$

or:
$$\frac{\frac{90}{229} - .25}{.75} = -0.1656(n - 1)^{0.537}$$
$$\log 0.188 = -0.1656(n - 1)^{0.537}$$
$$-0.725 = -0.1656(n - 1)^{0.537}$$

or: $\log 0.725 = 0.537 \log (n - 1) + \log 0.1656$

$-0.14 = 0.537 \log (n - 1) -0.78$

$0.64 = 0.537 \log (n - 1)$

$\log (n - 1) = 1.19$

$n - 1 = 15$

$n = 16$

∴ 16 load repetitions could be cycled at $60°F$ and 0.01 in./min. before failure.

(3) Determine the endurance limit of the mixture under the given test conditions ($60°F$, 0.01 in./min., unconfined condition).

$$x_c = x_0 \left[0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25 \right]$$

$$x_c = 229 \left[0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25 \right]$$

In this case, n gets large. The term $0.75 \cdot 10^{-0.1656(n - 1)^{0.537}}$ approaches 0. Thus, $x_c = 229 \, (0.25)$

$$x_c = 57 \text{ psi}$$

∴ the endurance limit at $60°F$, 0.01 in./min., unconfined is 57 psi.

DISCUSSION OF RESULTS

It was stated in the "Outline of Investigation" that the first objective of this study was to establish a relationship among the following variables: temperature, rate of deformation, and strength of a bituminous-aggregate mixture. The second objective was to study the effect of repeated loads since it was felt that there was some combination of applied stress and number of load applications that would lead to failure as defined by some suitable criterion. The third objective was to determine the effect of static loadings upon a compacted bituminous-aggregate mixture under different conditions of temperature.

This discussion of results will be presented under the following headings: 1) Compressive Strength Tests, 2) Unconfined, Repeated Load Tests, 3) Confined, Repeated Load Tests, and 4) Static Unconfined Load Tests.

Compressive Strength Tests

In evaluating the relative effects of the temperature and the rate of deformation upon the maximum compressive stress, Figures 19, 20, 21, 22, 23, and 24 show rather conclusively that a change in temperature has a much more pronounced effect upon the maximum compressive stress, either unconfined or confined, than does the rate of deformation. This was also indicated from the partial correlation coefficients discussed in a previous section where the correlation between the transformed

maximum compressive stress and the transformed rate of deformation was
0.53 while the correlation between the transformed maximum compressive
stress and the transformed temperature was a -0.82 for Mixture I tested
in the unconfined compression test.

It has been pointed out in other investigations (for more exact
details, see the section in the Review of Literature entitled "The Effect
of Temperature and Rate of Deformation upon Bituminous Mixture Properties")
that the strengths of all bituminous mixtures vary with temperature. At
normal temperatures an asphaltic mix is somewhat plastic. (A perfectly
plastic body must be loaded to some definite stress causing an extension
or contraction. When this stress is reached the substance starts to
deform permanently and continues as long as the stress is applied.) In
these tests, as the temperature was lowered for tests at a fixed rate of
deformation, a large increase in the maximum compressive stress was noted
in all cases. At 40°F the mix had lost much of its plastic character and
was behaving more like an elastic body in that an increase in stress
resulted in a proportional change in strain.

Increasing the rate of deformation had the same qualitative effect
upon the magnitude of the maximum compressive stress as lowering the
temperature, but to a lesser degree. A bituminous mixture has a resist-
ance to deformation which is called viscous resistance which is greatly
influenced by the rate of deformation. An example of the effect of the
rate of deformation upon the viscous resistance of an asphaltic mixture
is the analogy of striking the surface of water with the hand. If this
is done slowly the hand enters the water with little resistance. When
the hand is brought in contact with the water at a high rate of speed,
considerable resistance to entering the water is met.

The effect of confining pressures upon the maximum compressive stress at various temperatures and rates of deformation is shown in Figure 54. At 40°F, the confining pressure had practically no effect upon the maximum compressive stress at the rates of deformation of 0.02 or 0.2 in./min. This would indicate that the strength of the mixture at this temperature was due primarily to the binder. Figure 55 shows the Mohr circle diagram for the test condition of 0.2 in./min. and 40°F.

At a temperature of 140°F and a rate of deformation of 0.02 in./min., the confining pressure had a marked effect upon the maximum compressive stress. The increase from 0 psi confining pressure, or the unconfined state, to 15 psi confining pressure had a much greater effect upon the maximum compressive stress than the increase from 15 psi confining pressure to 30 psi confining pressure. It would appear that the effect of the confining pressure upon the maximum compressive strength tends to level off as the confining pressure is increased. Figure 56 shows the Mohr circle diagram for the test condition of 0.2 in./min. and 140°F.

These results have been interpreted in terms of normal stress envelopes as discussed by Wilson [44]. The Mohr theory of failure has been used to evaluate the failure of bituminous mixtures. The Mohr theory states that there is a combination of normal stress and shearing stress in a finite system that will result in failure when a certain value. The failure relationship between normal and shear stress is known as Mohr's circle. An envelope of failure rupture can be drawn by using various confining pressures and the maximum compressive stress for those conditions. A line drawn tangent to these

RELATIONSHIP BETWEEN MAXIMUM COMPRESSIVE STRESS

AND THE CONFINING PRESSURE FOR

VARIOUS RATES OF DEFORMATION AND TEMPERATURES
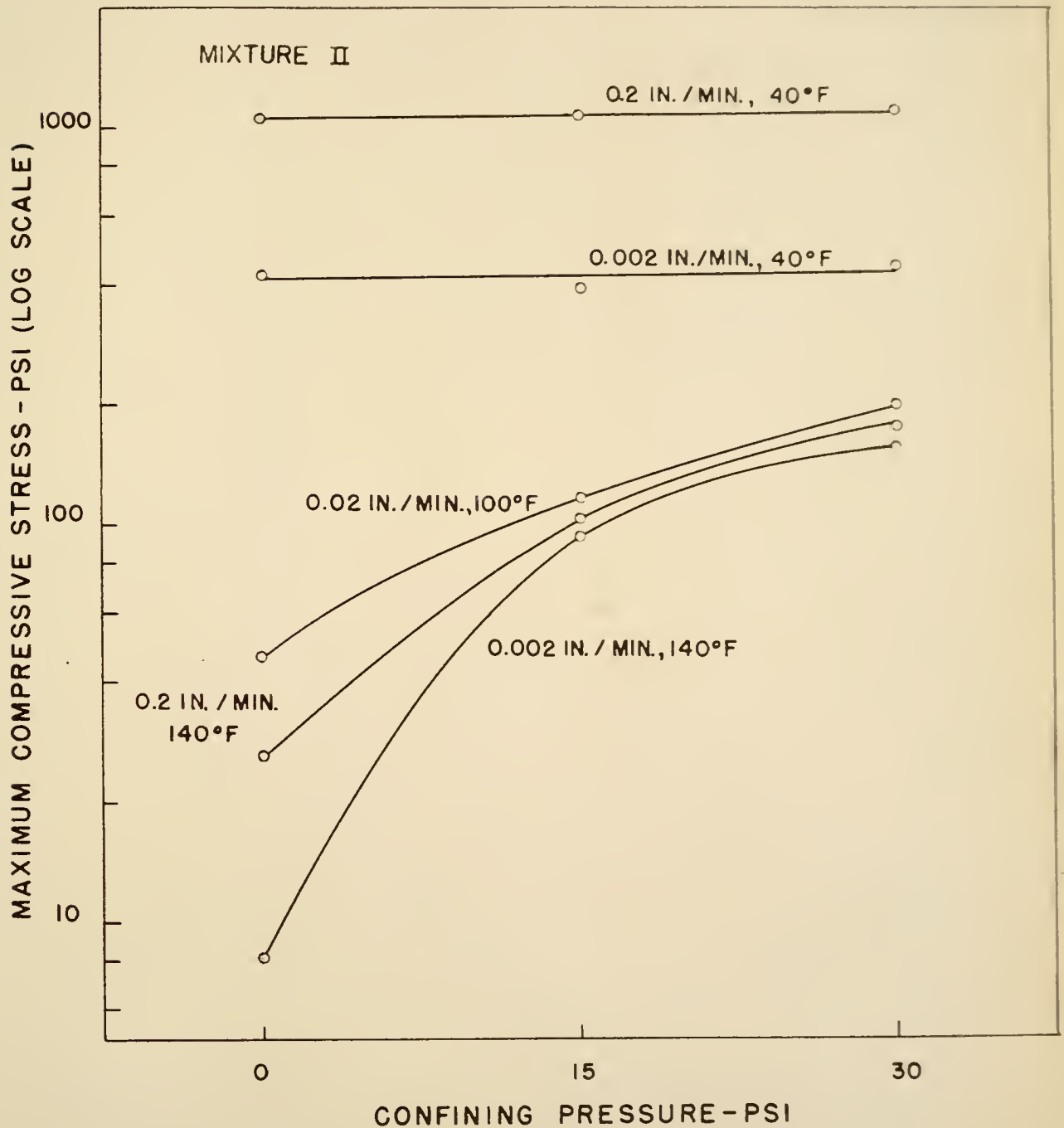
FIGURE 54

MOHR CIRCLE DIAGRAM FOR
SAND-ASPHALT MIXTURE

MIXTURE II
0.2 IN./MIN., 40°F

FIGURE 55

NORMAL STRESS, LBS./SQ IN.

SHEAR STRESS, LBS./SQ. IN.

φ = 11.5

C = 435 PSI

MOHR CIRCLE DIAGRAM FOR
SAND-ASPHALT MIXTURE

MIXTURE II
0.002 IN./MIN., 140°F

φ = 38 7
C = 7 PSI

NORMAL STRESS, LBS./SQ.IN.

SHEAR STRESS, LBS./SQ.IN.
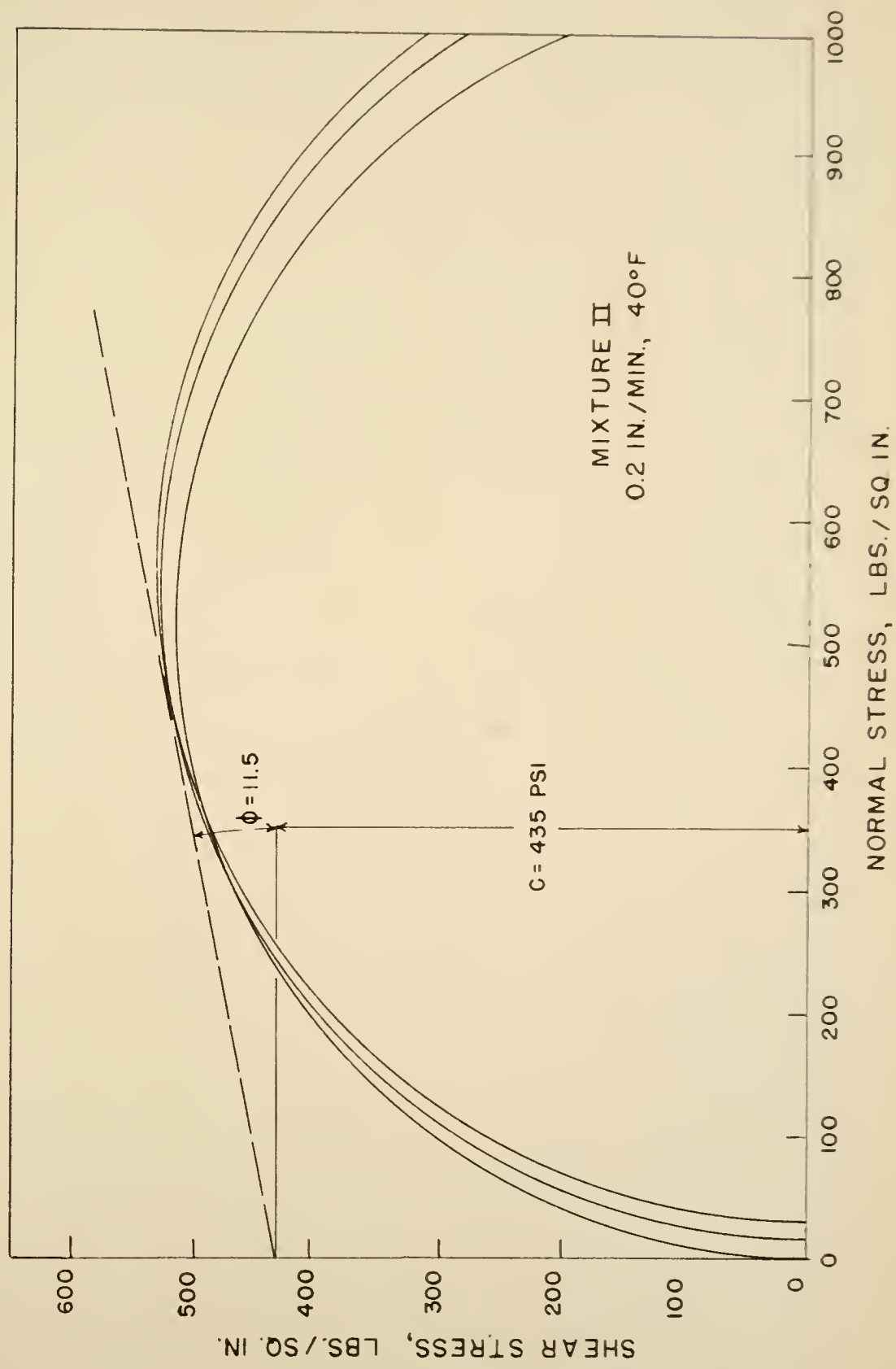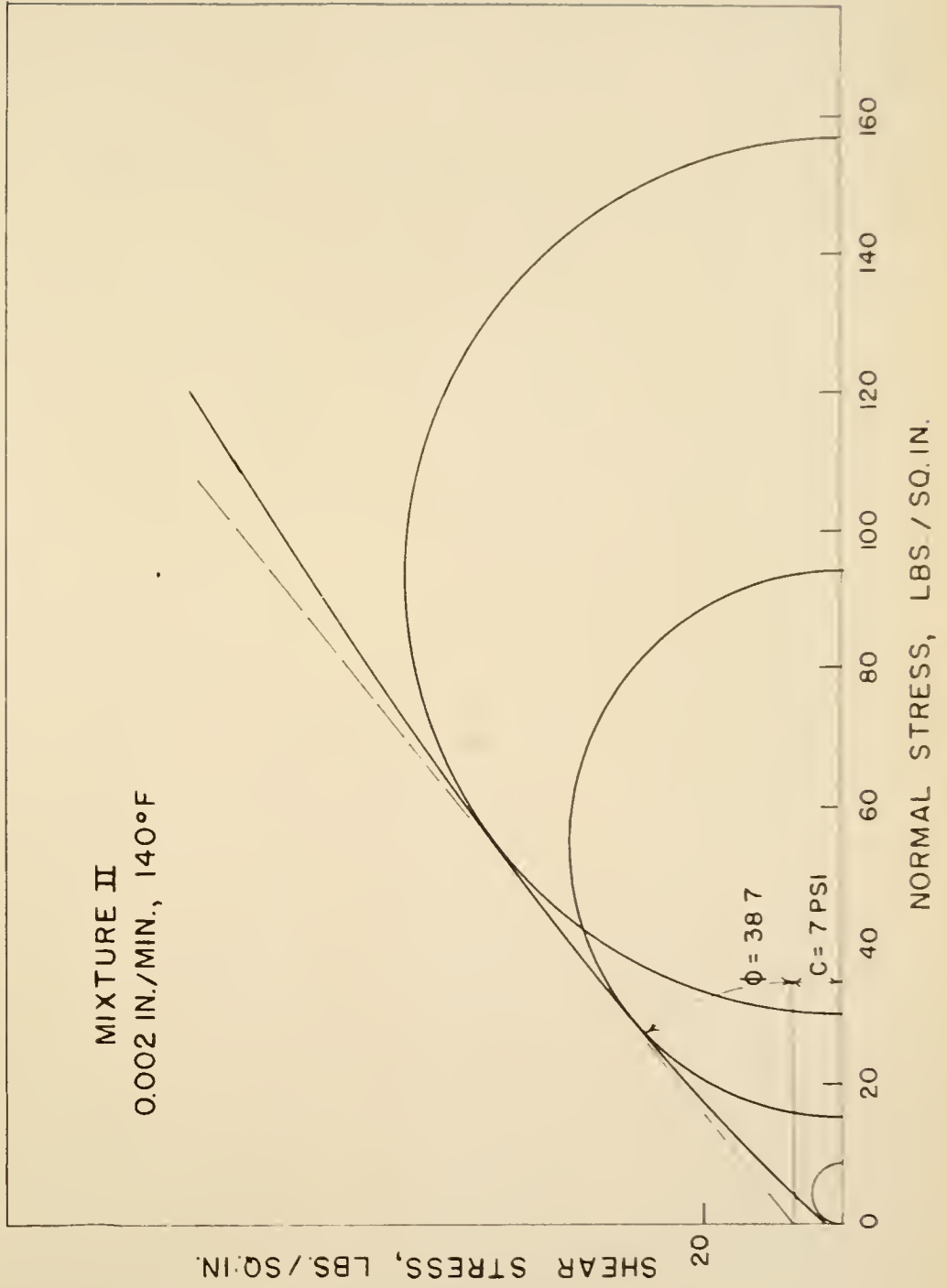
FIGURE 56

circles is called the Mohr rupture envelope. This envelope represents the combination of shearing and normal stresses that results in failure. McLeod (56) states that the curved Mohr envelope is a fundamental and not an accidental characteristics of bituminous mixtures.

Unconfined, Repeated Load Tests

The effects of repeated loads at various percentages of the maximum unconfined compressive stress (dependent upon the test conditions) are shown graphically in Figures 25 through 33. It can be noted that the relationship starts out as a straight line in all cases when the permanent deformation is plotted against the logarithm of the number of load repetitions. At some stage, dependent upon the applied stress, the plot deviates sharply from the straight line, as is shown in the upper two curves of all the figures. What occurs at this point is a matter of conjecture. It is hypothesized that the asphalt film between aggregate particles is being reduced in dimension until some critical thickness is reached. At this point, in order to sustain the load, adjustment in the specimen takes place by reorientation of the aggregate particles themselves. This gives rise to excessive shear deformations which are measured as permanent deformations. The point where excessive shear deformation occurs was taken as the failure criterion.

Under this hypothesis, the elastic part of the deformation would take place principally in the asphalt film which is bound firmly to the aggregate in a polymolecular layer. As the applied stress decreased, the number of loading cycles necessary to cause deviation from a straight line plot of deformation versus log of load repetitions increased. When the applied stress was 25 percent of the maximum compressive stress, no

deviation was observed for the number of load repetitions used in this sequence of observations. At 50 percent of the maximum compressive stress excessive shear deformations were noted in all cases. This indicates that the endurance limit lies between 25 and 50 percent. No attempt was made to determine the endurance limit more closely.

A graphical representation of the deformations experienced by a specimen subjected to unconfined repeated loads at a temperature of $140^{\circ}F$ and at a rate of deformation of 0.2 in./min. is shown in Figure 57. The applied stress in this test was 13 psi. The arrows indicate the deformation record of the specimen while being loaded and unloaded. An important point brought out by this plot is that the elastic rebound appears to be independent of the past deformations the specimen has undergone. The dotted parallel lines show this. These two lines are the accumulated permanent deformation line and the accumulated total deformation line. Even after excessive shear deformations have taken place, the elastic rebound appears to remain constant. This tends to support the concept that the elastic part of the deformation takes place principally in the asphalt films surrounding the aggregate particles.

The curves shown in Figure 58 represent the break-down of the deformation experienced by a specimen for each load cycle in the repeated load test at 0.2 in./min. and $140^{\circ}F$ using an applied stress of 13 psi. It shows the elastic rebound as a constant for the duration of the test. The amount of permanent deformation experienced by the specimen for each load cycle decreased to a minimum point and then increased rather sharply. It can be seen upon examination of the accumulated permanent deformation curve that this sharp increase in permanent deformation per cycle results

DEFORMATION RECORD

OF A SPECIMEN SUBJECTED

TO REPEATED LOAD

FIGURE 57
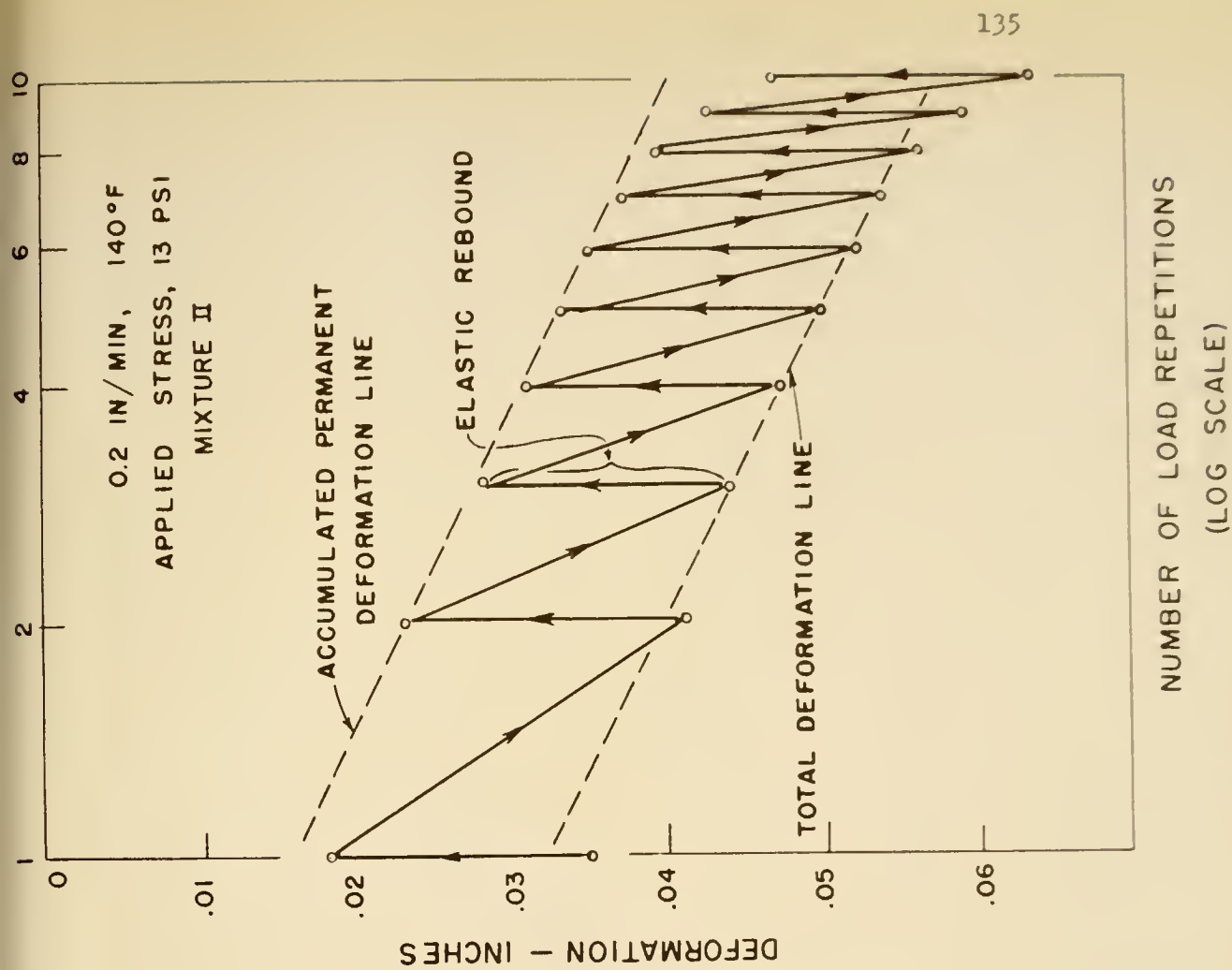
RELATIONSHIP BETWEEN

ELASTIC AND PERMANENT

DEFORMATION

WITH REPETITIVE LOADING
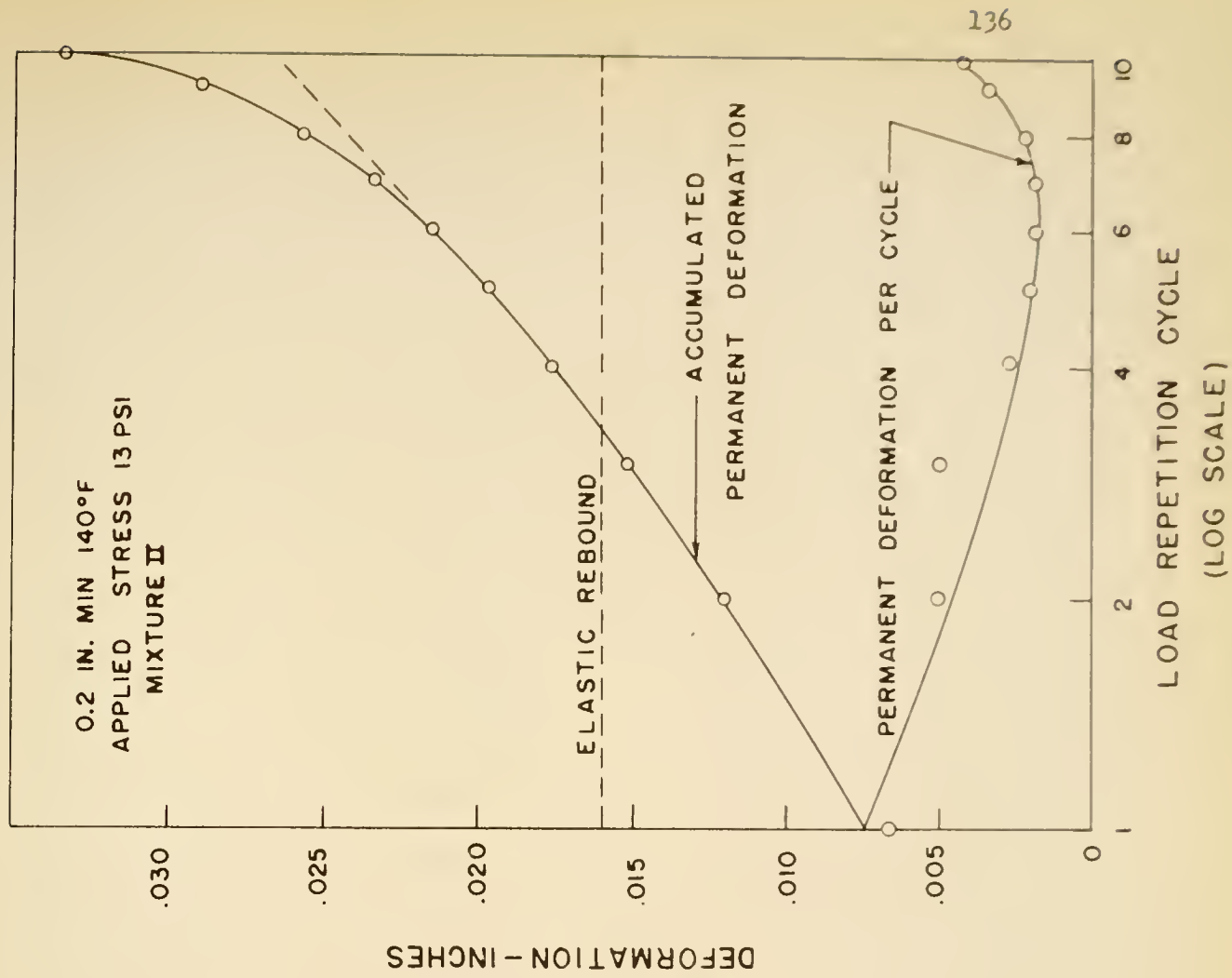
FIGURE 58

in the deviation from a straight line plot. It was postulated that excessive shear deformations became important at this (failure) point.

Figure 34 shows the relationship between the applied stress and the number of unconfined load repetitions a specimen was able to withstand at 140°F and varying rates of deformation before an excessive shear deformation, which was defined as a failure criterion, was observed. The point where the permanent deformation per cycle curve swings upward in Figure 58 would show in Figure 34 on the 0.2 in./min. plot at 13 psi and 7 cycles. Under each test condition there appeared to be an applied stress that could be cycled without excessive shear deformations occurring. At some levels no further increase in permanent deformation was noted after a small number of load applications. This stress in each case was labeled as the endurance limit. At each test level the endurance limit was approximately 25 percent of the maximum compressive stress. The endurance limits at 140°F for 0.2, 0.02, and 0.002 in./min. are shown in Figure 34 as 7 psi, 4 psi, and 2 psi respectively. The asphalt film must have been able to absorb these stresses since apparently no particle reorientation occurred. It can be noted that a large change in the rate of deformation did not bring about too great a change in the endurance limit.

The effect of temperature on the applied stress-number of load repetitions relationship for one rate of deformation, 0.2 in./min., is shown in Figure 35. Here it can be seen that a change in temperature resulted in a large change in the endurance limit. The endurance limits at 0.2 in./min. for 40°, 100°, and 140°F are shown as 259 psi, 25 psi, and 7 psi respectively. Again, it can be seen, in Figure 35, that a

decrease in the applied stress permitted a specimen to withstand a greater number of load repetitions before the failure criterion was reached.

Figure 36 enables one to make a comparison of the relative effects of the rate of deformation and temperature upon the endurance limit. It can be seen that the temperature has a much greater effect upon the endurance limit than does the rate of deformation. For example, at a temperature of 40°F changing the rate of deformation from 0.2 in./min. to 0.002 in./min. lowers the endurance limit from 259 psi to 100 psi. At a rate of deformation of 0.2 in./min. changing the temperature from 40°F to 140°F lowers the endurance limit from 259 psi to 7 psi.

The concept has been presented that the elastic part of the deformation takes place principally in the asphalt film surrounding the aggregate particles. The endurance limit and the elastic part of the deformation must be closely related. Under this hypothesis a certain film thickness must be present to ~~absorb~~ resist the stress known as the endurance limit.

It would appear that the effective thickness of this asphalt film is associated with the viscous resistance which in turn is affected by temperature and rate of deformation. The viscous resistance of the asphalt film varies directly with the rate of deformation and inversely with the temperature.

Confined, Repeated Load Test

The effect of lateral support upon the relationship between applied stress, number of load applications necessary to reach failure, tempera-

ture, and rate of deformation are shown in Figures 37 through 46. It
can be seen by comparing Figures 36 and 47 that for any one temperature
and rate of deformation applying a a confining pressure increases the
stress a specimen can stand without failing during a repeated load test.
The confined, repeated load curves are quite similar to the unconfined
repeated load curves for plots of permanent deformation versus the log
of the number of load repetitions. This can be seen by comparing
Figures 36 and 47.

In the confined repeated load test the applied loads were 50 per-
cent and 25 percent of the maximum compressive stress as determined for
the fixed confining pressure, temperature, and rate of deformation.
With the knowledge gained by the more complete study in the unconfined,
repeated load sequence, it was felt that there was no need for performing
tests at the 75 percent level. The main item to be evaluated was the
stress the specimen could withstand without reaching the failure cri-
terion under a repetition of load applications. The failure criterion
was discussed in a previous section, "Unconfined, Repeated Load Test."

Figures 37 through 46 indicate that an applied stress of 25 percent
of the maximum compressive stress for each given test condition (repre-
sented by the lower line in each figure) could be cycled a number of
times without excessive shear deformations occurring. This value is
that referred to in a previous section as the endurance limit. Thus,
it can be seen that, for the mixture being tested (Mixture II), the
endurance limit was approximately 25 percent of the maximum compressive
stress determined at a given set of test conditions regardless of the
lateral support applied.

This is important since it shows that the addition of lateral
support doesn't change the basic behavior of the mixture in any unex-
pected manner. While it was stated that the endurance limit remained
at about 25 percent of the maximum compressive stress, it must be remem-
bered that at the lower rates of deformation and the higher temperatures,
introducing lateral support materially increased the maximum compressive
strength of the mixture. Thus, even though the endurance limit was
approximately 25 percent of the maximum compressive stress for both the
unconfined and the confined condition, the endurance limit for the con-
fined case was numerically much higher for the high temperatures and low
rates of deformation than for the unconfined case. A comparison of
Figures 36 and 47 will graphically show this.

Figure 47 shows the effect of lateral support upon the applied
stress-endurance limit at various temperatures and rates of deformation.
At a temperature of 40°F and a rate of deformation of 0.2 in./min.,
increasing the confining pressure from 0 psi to 15 psi and then to 30
psi increased the endurance limit from 260 psi, to 270 psi, and to 275
psi respectively. Thus it can be seen that the introduction of lateral
support at that test level did not affect the endurance limit appreciably.
At that temperature and rate of deformation the viscosity of the binder
is such that the introduction of a confining pressure in the ranges of
0-30 psi had little effect upon the endurance limit. One wouldn't expect
the introduction of lateral support to affect the endurance limit at
40°F since there was little increase in the maximum compressive stress
between 0 and 30 psi, at that temperature.

At the other extreme of test conditions, a temperature of 140°F and a rate of deformation of 0.002 in./min., increasing the confining pressure from 0 psi to 15 psi to 30 psi increases the endurance limit from 2 psi to 23 psi to 29 psi. This can be seen in Figure 47. In this case the confining pressure had a marked effect upon the endurance limit. At a temperature of 140°F and 0.002 in./min., the viscosity of the binder was so low that the introduction of the confining pressure "stiffened" the mix appreciably.

Conditions of loading for a bituminous-aggregate layer in the field are such that a certain degree of confinement is present under a loaded area. McLeod (56) hypothesized that the degree of confinement in a bituminous pavement has a minimum value of the unconfined compressive stress of the paving mixture. The unconfined compressive stress of the mixture being investigated was 8 psi at a temperature of 140°F and a rate of deformation of 0.002 in./min. The endurance limit for these conditions was given previously as 2 psi. With a confining pressure of 15 psi the endurance limit had increased markedly to 23 psi. These results indicate that conditions in the field are such that a mixture could be provided with sufficient lateral support to resist rutting tendencies.

Static Unconfined Load Tests

The effect of a static load upon a compacted bituminous specimen tested in the unconfined condition at five temperatures, 40, 55, 70, 100, and 140°F, is shown in Figures 48 through 52. These curves show the deformation-time relationship at different temperatures for various applied stresses. The deformation that a specimen undergoes was sub-

divided by Mack (46) into three parts: 1) an instantaneous elastic deformation independent of time, 2) a retarded elastic strain which is a function of time, and 3) a plastic strain whose rate decreases with time--in the case where the specimen withstands the applied stress.

The data from the static unconfined load tests show again that temperature had a great influence upon the maximum stress that a specimen could withstand. Figure 53 shows the relationship between maximum static stress and temperature. The change of the character of the mixture with temperature is shown quite plainly by comparing the time-deformation curve for 40°F as shown in Figure 52 with the time-deformation curve for 140°F as shown in Figure 48. At 140°F the mixture was quite plastic in character as evidenced by the curve for the applied stress of 20 psi. Between 150 and 200 seconds the amount of deformation increased rapidly from a value of 0.05 inch to a condition where failure was imminent and the load had to be removed immediately in order to obtain a rebound value. At 40°F the mixture exhibited considerable viscous resistance. The curve of the applied stress of 375 psi was sloping upward gradually even after a deformation of almost 0.2 inches during a time interval of 2,500 seconds.

When comparing results of the static unconfined load test with the results of the unconfined, repeated load test, it would appear that the unconfined, repeated load test is more severe. For example, the endurance limit for 40°F and a rate of deformation of 0.002 in./min. (the slowest rate) was 105 psi, while at 40°F the maximum static stress was 350 psi; for 100°F and a rate of deformation of 0.002 in./min. the endurance limit was 8 psi, while at 100°F the maximum static stress was 20 psi;

for 140°F and a rate of deformation of 0.002 in./min. the endurance
limit was 2 psi while at 140°F the maximum compressive stress was 10 psi.
The failure criterion which was evolved for the unconfined, repeated
load test was more severe in mixture evaluation than was the failure cri-
terion established for the static unconfined load test. Excessive shear
deformations were noted in the unconfined, repeated load test before a
large amount of aggregate particles were displaced. Therefore the
cross sectional area of the specimen under test did not enlarge to any
extent. In the case of the static load test, some specimen bulging was
noted which meant that the cross sectional area of the specimens under
test was increasing. This meant that although the applied load remained
constant during the test the applied stress was decreasing. This fact
was not taken into account in determining the maximum static stress.

SUGGESTIONS FOR FURTHER RESEARCH

Since all of the work in this study on repeated loads was on a
sand-asphalt mixture, it would be of much interest if a study were made
on the effect of repeated loads upon mixtures that had a wide range of
gradation and aggregate size.

Additional study could be performed with the sand-asphalt mixture
to evaluate the effect of asphalt content upon the endurance limit.
While in the case of the asphalt content (9 percent) used in this study,
the endurance limit was approximately 25 percent of the maximum compres-
sive stress, it might be that a higher asphalt content would result in
an endurance limit greater than 25 percent.

All tests were performed in this study on specimens whose height to
diameter ratio was such that the end effects caused by friction between
the specimen and the loading head were minimized. A closer approximation
of evaluating actual field conditions could be made by performing repeated-
load tests on specimens whose height to diameter ratio was such that the
specimens would be considered irrational. An extension of this idea
would be to perform repeated load tests on irrational specimens where
the load is applied over an area somewhat less than the cross sectional
area of the specimen.

In the final analysis, no laboratory study is complete in itself.
Correlation of a laboratory study with field performance is the ultimate
aim. Therefore, it would be very desirable to establish the maximum

stress and the number of applications of that stress to which a bitu-
minous overlay showing signs of distress had been subjected; and to see
if, in the laboratory, these conditions could be duplicated. If this
could be done one would be in a position to evaluate mixtures in a
manner which is now not possible.

CONCLUSIONS

It must be remembered that this was a laboratory study. The majority of tests were performed with an aggregate of a sheet-asphalt gradation and the resulting mixtures were more plastic in character than any bituminous concretes. Tests were performed in both the unconfined and confined state. In the field, it is recognized that mixtures are loaded in such a manner that some degree of confinement exists. However, since the degree of confinement is unknown the lateral support provided for in the laboratory tests may not have been of the proper magnitude to simulate possible field conditions.

With these limitations in mind, the following conclusions are presented:

1. The effect of temperature upon the maximum compressive stress and the endurance limit was more pronounced for the test conditions used than was the effect of the rate of deformation.

2. The relationship developed among the variables of maximum compressive stress (confined or unconfined), temperature and rate of deformation can be used to obtain information concerning the strength of the mixture under many combinations of test conditions. It appears that this relationship can be established with results obtained under a limited number of test conditions.

3. The general relationship established among maximum compressive stress, temperature, and rate of deformation by means of the unconfined

compression test appeared to hold true for the same variables when the test was conducted with varying degrees of lateral support.

4. For the sand-asphalt mixture, the effect of the confining pressure upon the maximum compressive stress tends to level off as the confining pressure is increased. Thus, a curved Mohr envelope of failure results.

5. There was found to be a combination of applied stress and number of load repetitions that resulted in excessive shear deformations. These excessive shear deformations appear in a plot of deformation versus the logarithm of the number of load repetitions as a deviation from a straight line.

6. Repetitive load test results suggest that the asphalt film between particles was reduced in dimension until some critical thickness was reached. At this point, in order to sustain the load, the adjustment in the specimen takes place by reorientation of the aggregate particles themselves. This particle reorientation results in permanent deformation.

7. Limited results indicate that the elastic part of the deformation takes place principally in the asphalt film which is bound firmly to the aggregate particle in a polymolecular layer.

8. Under each test condition, there appeared to be an applied stress that could be cycled without excessive shear deformations occurring. This stress was labeled as the endurance limit.

9. The use of lateral support up to 30 psi increased, markedly, the maximum compressive stress at high temperatures. The rate of deformation had a more pronounced effect upon the maximum confined compressive stress at $140^{\circ}F$ than at $40^{\circ}F$.

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BIBLIOGRAPHY

10. For the mixtures tested in this study, it was found that the endurance limit was approximately 25 percent of the maximum compressive stress for all test conditions used. This endurance limit appears to be an important mix property.

11. The failure criterion which was evolved for the unconfined, repeated load test was more severe in mixture evaluation than was the failure criterion established for the static, unconfined load test. This resulted in the conclusion that the static, unconfined load test was a less rigorous test for the bituminous mixtures tested than was the unconfined, repeated load test.

12. It appears that a promising means of evaluating the adequacy of a bituminous mixture for utilization in the field is to perform the confined, repeated load test on a rational specimen at a temperature of 140°F and a rate of strain of 0.0005 in./in./min. The degree of confinement to be introduced can not yet be recommended. From previous work on the strength of bituminous mixtures, it is known that some degree of confinement should be used in order to distinguish those mixtures whose strengths are particularly benefited by confinement.

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A P P E N D I X A

Hubbard-Field Test Data

APPENDIX A

Hubbard-Field Test Data

Asphalt, % Agg. Wgt.	7	8	9	10	11
% Aggregate Voids	27.6	26.0	26.3	29.3	31.5
% Voids – Total Mix	31.1	9.6	7.1	6.8	5.6
Unit Wgt. ($\#/ft.^3$)	131.3	136.1	136.8	133.7	131.4
Stability (#)	1600	1900	2000	1100	1000
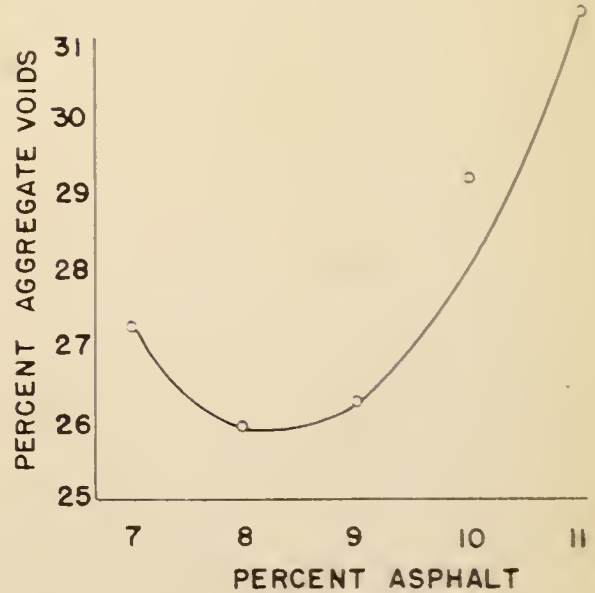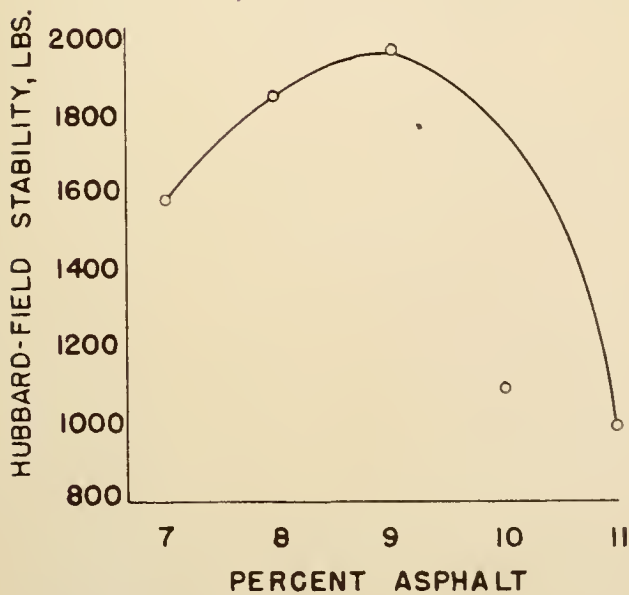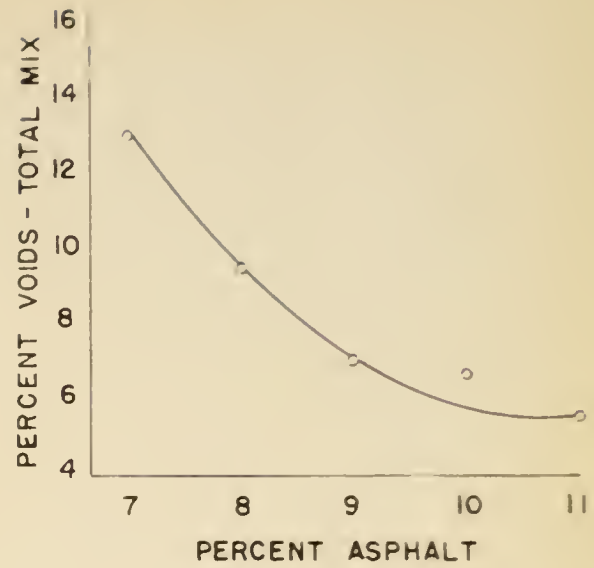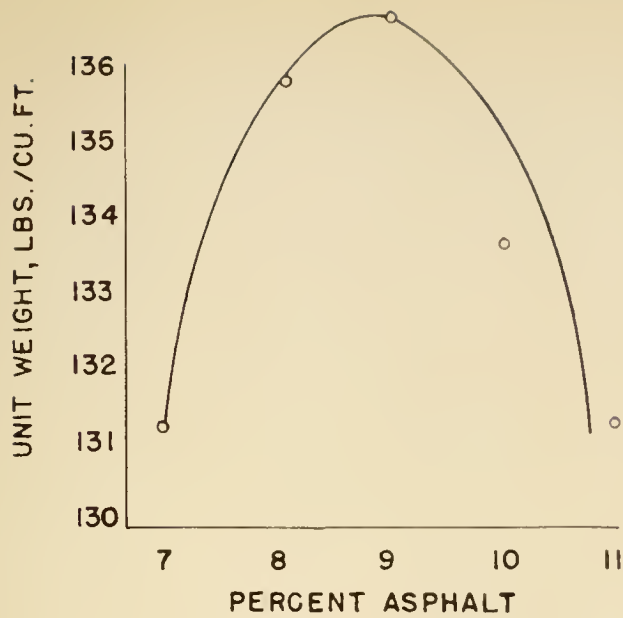
HUBBARD-FIELD TEST PROPERTIES

VERSUS

ASPHALT CONTENT BY WEIGHT OF AGGREGATE

FIGURE 59

A P P E N D I X B

Multiple Linear Regression Analyses

Unconfined Compression Test Data, Mixture I

Key to Coded and Transformed Data.

$$x_0 = 10000 \log \log x_0 \qquad\qquad x_2 = x_2/20$$

$$x_1 = 1000 \log 1000 \ x_1 \qquad\qquad x_3 = 50 \ x_2 \log 1000 \ x_1$$

where x_0 = maximum unconfined compressive stress, psi

x_1 = rate of deformation, in./min.

x_2 = temperature, ^{o}F

The multiple linear regression equation is as follows:

$$x_0 = B_0 + B_1 X_1 + B_2 X_2 + B_3 X_3 \quad \cdots \quad \cdots \quad (1)$$

where B_0, B_1, B_2, and B_3 are the unknown parameters.

Equation 1 may be written in terms of deviations from means.

$$(X_0 - \bar{X}_0) = \quad + B_1(X_1 - \bar{X}_1) + B_2(X_2 - \bar{X}_2) + B_3(X_3 - \bar{X}_3) \quad \cdots \quad (2)$$

If both sides of the equation are divided by the variance of X_0 and

each term of the independent variables is multiplied and divided by

its variance one gets the standard data form:

$$\frac{X_0 - \bar{X}_0}{\sigma_{X_0}} = \frac{\sigma_1 B_1}{\sigma_{X_0}} \frac{(X_1 - \bar{X}_1)}{\sigma_1} + \frac{\sigma_2 B_2}{\sigma_{X_0}} \frac{(X_2 - \bar{X}_2)}{\sigma_2} + \frac{\sigma_3 B_3}{\sigma_{X_0}} \frac{(X_3 - \bar{X}_3)}{\sigma_3} \quad \cdots \quad (3)$$

Letting $\dfrac{(X_i - \bar{X}_0)}{\sigma_{X_i}} = Z_i$ and $\dfrac{\sigma_{X_i}}{\sigma_{X_0}} B_i = B_i'$ one gets:

$$Z_0 = B_1' Z_1 - B_2' Z_2 - B_3' Z_3 \quad \cdots \quad \cdots \quad (4)$$

The problem now is to obtain the best sample estimates b_1', b_2', and

b_3' of B_1', B_2', and B_3' such that the sum of squares of the deviations

is a minimum. This is done by a least squares analysis using correla-

tion coefficients r_{ij}'s (for a definition of correlation coefficients,

see section on Regression Analysis and Statistical Inferences) as a
simplified means of calculation. The calculations themselves follow
and equations used in the analysis are presented.

Observation	X_0	X_1	X_2	X_3
1	4233	301	2	602
2	1659	301	5	1505
3	72	301	7	2107
4	4486	1301	2	2602
5	2659	1301	5	6505
6	1316	1301	7	9107
7	4783	2301	2	4602
8	3222	2301	5	11505
9	2051	2301	7	16107
10	5072	3301	2	6602
11	3800	3301	5	16505
12	2848	3301	7	23107
13	5172	3987	2	7874
14	4036	3987	5	19685
n = 15	3131	3987	7	27559

	X_0	X_1	X_2	X_3
Σu = sums	48540	33423	70	155974
$\bar{u}$ = means	3236	2228.2	4.666	10398.266

		X_0	X_1	X_2	X_3
	X_0	188,185,390	123,208,144	190,298	506,740,174
Σuv	X_1		100,423,119	155,974	468,641,222
	X_2			390	868,998
	X_3				2,611,001,094

		X_0	X_1	X_2	X_3
	X_0	157,075,440	108,156,828	226,520	504,731,864
$\frac{\Sigma u \Sigma v}{n}$	X_1		74,473,128.6	155,974	347,541,266
	X_2			326.667	727,878.67
	X_3				1,621,859.25

		X_0	X_1	X_2	X_3
Sums of	X_0	31,109,950	15,031,316	-36,222	2,008,310
Product	X_1		25,949,990	0	121,099,955
SP(u,v)	X_2			63.33	141,119
	X_3				989,141,849

where $\Sigma u = \Sigma Xij$ $\qquad \Sigma uv = \Sigma Xij \cdot Xij$

$i = 0, 1, 2, 3 \quad \Sigma u \Sigma v = \dfrac{(\Sigma Xij)(\Sigma Xij)}{n}$, SP(u, v) $= \Sigma uv - \dfrac{(\Sigma u \Sigma v)}{n}$

$j = 1, \text{------} n$

The correlation coefficients, the r_{ij}'s, are calculated from the formula:

$$r_{ij} = \frac{SP(X_i X_j)}{\sqrt{SP(X_i X_i)} \; SP(X_j X_j)}$$

For this analysis:

$r_{01} = 0.5297$ $r_{12} = 0$

$r_{02} = -0.8182$ $r_{13} = 0.7559$

$r_{03} = 0.0114$ $r_{23} = 0.5653$

From the above results one can establish a system of three linear equations and three unknowns. (See page 205 of Reference 55.)

$$1 \; b_1' + \quad 0 \; b_2' + 0.7559 \; b_3' = 0.5297$$

$$0 \; b_1' + \quad 1 \; b_2' + 0.5653 \; b_3' = 0.8182$$

$$0.7559 \; b_1' + 0.5653 \; b_2' + \quad 1 \; b_3' = 0.0114$$

$$b_1' = 0.0205, \; b_2' = 1.1990, \; b_3' = 0.6737$$

Including those values in equation 4 one obtains:

$$z = 0.0205 \; \frac{(X_1 - \bar{X}_1)}{\sigma X_1} - 1.199 \; \frac{(X_2 - \bar{X}_2)}{\sigma X_2} + 0.6737 \; \frac{(X_3 - \bar{X}_3)}{\sigma X_3}$$

By making the proper substitutions in the above expression one can obtain equation 1 as follows:

$$X_0 = 5876.33 + 0.0224 \; X_1 - 842 \cdot 69 \; X_2 + 0.1195 \; X_3$$

Sum of squares due to regression $= b_1 SP(X_1 X_0) - b_2 SP(X_2 X_0) - b_3 SP(X_3 X_0)$

$$SSR = 0.0224 \; (15,051,316) - 842.69 \; (-36,222) + 0.1195 \; (2,008,310)$$

$$= 31,101,059.7$$

Total Sum of Squares $= 31,109,950$

R^2 has been defined as the proportion of the sum of squares of the dependent variable which is explained by the multiple regression equation.

$$R^2 = \frac{\text{sum of squares attributable to regression}}{\text{total sum of squares}}$$

For this analysis: $R^2_{0.123} = \frac{31,101,059}{31,109,950} = 0.997$

Also $R^2_{0.123} = r^2_{01} + r^2_{02} + r^2_{03}$

This relationship was used to determine the effect of dropping out variables from the study. Thus:

$R^2_{0.12} = (0.5297)(0.5297) + (0.8182)(0.8182) = 0.95$

and $R^2_{0.1} = (0.5297)(0.5297)$ $= 0.28$

Unconfined Compression Test Data, Mixture II

All that was desired in this study was to obtain the raw data prediction equation from a limited number of results. The calculation and procedure are the same as for Mixture I.

Key to Coded and Transformed Data.

$$X_0 = \log \log 10 \, x_0 \qquad\qquad X_2 = x_2$$

$$X_1 = \log 1000 \, x_1 \qquad\qquad X_3 = x_2 \log 1000 \, x_1$$

		X_0	X_1	X_2	X_3
		.5591	.301	40	12.04
		.2795	.301	140	42.14
		.6307	2.301	40	92.04
		.3829	2.301	140	322.14
	$\bar{u}$	.4563	1.301	90	117.09
	$\Sigma u =$	1.8252	5.204	360	468.36
Σuv	X_0	.9017792	2.522585	139.248	197.42165
	X_1		10.7704	468.36	969.33636
	X_2			42400.	55162.4
	X_3				114,166.2824
$\dfrac{\Sigma u \Sigma v}{n}$	X_0	.832838	2.374585	164.268	213.712668
	X_1		6.770404	468.360	609.33636
	X_2			32400.	42152.40
	X_3				54,840.272
$SP(u,v)$	X_0	.0689412	.148	-25.02	-16.291018
	X_1		4.0000	0	360.0000
	X_2			10000.	13010.
	X_3				59,326.01

In this analysis, the Sum of Products (u,v) will be used rather than the r_{ij}'s used previously. (See page 204, Reference 55, for the equations generated.)

This gives the following three equations and three unknowns:

$$4\ b_1 + \quad 0\ b_2 + \quad 360\ b_3 = 0.148$$

$$0\ b_1 + 10,000\ b_2 + 13,010\ b_3 = -25.02$$

$$360\ b_1 + 13,010\ b_2 + 59,326.01\ b_3 = -16.291018$$

Solving these equations one obtains: $b_1 = 0.0105$, $b_2 = -0.00288$,

$b_3 = 0.000294$.

This gives: $X_0 = 0.4563 + 0.0105\ (X_1 - 1.301) - 0.00288\ (X_2 - 90) +$

$\qquad$ $0.000294\ (X_3 - 117.09)$.

Collecting terms: $X_0 = 0.667414 + 0.0105\ X_1 - 0.00288\ X_2 + 0.000294\ X_3$

or $\quad$ log log 10 $X_0 = 0.667414 + 0.0105$ log 1000 $X_1 - 0.00288\ X_2 +$

$\qquad$ $0.000294\ X_2$ log 1000 X_1

Substituting in this equation, the maximum unconfined compressive stress

is predicted for the following test conditions:

$$40^{\circ}\text{F} \qquad 0.02\ \text{in./min.} \ = \ 650\ \text{psi}$$

$$(\ 0.002\ \text{in./min.} \ = \ \ 29\ \text{psi}$$

$$100^{\circ}\text{F} \ \{ \ 0.02\ \text{in./min.} \ = \ \ 50\ \text{psi}$$

$$(\ 0.2\ \text{in./min.} \ \ \ = \ \ 91\ \text{psi}$$

$$140^{\circ}\text{F} \qquad 0.02\ \text{in./min.} \ = \ \ 14\ \text{psi}$$

Unconfined Compression Test
Indiana AH Type B Surface Mixture

All that was desired in this study was to obtain the raw data prediction equation from a limited number of results. The calculations and procedures are the same as for Mixture I.

Key to Coded and Transformed Data.

$$X_0 = \log \log x_0 \qquad X_2 = x_2$$

$$X_1 = \log 1000 \, x_1 \qquad X_3 = x_2 \log 1000 \, x_1$$

	X_0	X_1	X_2	X_3
	.42004	.301	40	12.04
	.17064	.301	140	42.14
	.47978	2.301	40	92.04
	.26661	2.301	140	322.14
Σu	1.33707	5.204	360	468.36
$\bar{u}$	.33343	1.301	90	117.09

		X_0	X_1	X_2	X_3
Σuv	X_0	.506821	1.89524	97.2078	142.29275
	X_1		10.7704	468.36	609.33636
	X_2			42400.	55162.40
	X_3				114,166.2824

		X_0	X_1	X_2	X_3
$\dfrac{\Sigma u \Sigma v}{n}$	X_0	.446939	1.739528	120.3363	156.55753
	X_1		6.7704	468.36	609.33636
	X_2			32400.	42152.40
	X_3				54840.272

		X_0	X_1	X_2	X_3
$SP(u,v)$	X_0	.059882	.155712	-23.1285	-14.26478
	X_1		4.0000	0	360.00
	X_2			10000.	13010.
	X_3				59326.01

This gives the following three equations and three unknowns:

$$4 \, b_1 + \qquad 0 \, b_2 + \qquad 360 \, b_3 = 0.1557$$

$$0 \, b_1 + 10,000 \, b_2 + 13,010 \, b_3 = -23.129$$

$$360 \, b_1 + 13,010 \, b_2 + 59,326.01 \, b_3 = -14.265$$

Solving these equations, one obtains: b_1 = .02264, b_2 = .002548,

b_3 = .00018097.

This gives: X_0 = 0.33343 + 0.02264 (X_1 - 1.301) - 0.002548 (X_2 - 90) +

0.00018097 (X_3 - 117.09).

Collecting terms: X_0 = 0.51201 + 0.02264 X_1 - 0.002548 X_2 + 0.00018097 X_3.

Substituting in this equation, the maximum confined compressive

stress is predicted for the following test conditions:

$$40^\circ F \quad 0.02 \text{ in./min.} \quad = \quad 648 \text{ psi}$$

$$100^\circ F \quad \begin{cases} 0.002 \text{ in./min.} & = & 72 \text{ psi} \\ 0.02 \text{ in./min.} & = & 110 \text{ psi} \cdot \\ 0.2 \text{ in./min.} & = & 175 \text{ psi} \end{cases}$$

$$140^\circ F \quad 0.02 \text{ in./min.} \quad = \quad 45 \text{ psi}$$

Confined Compression Test Data, Mixture II - 15 psi

All that was desired in this study was to obtain the raw data prediction equation from a limited number of results. The calculations and procedures are the same as for Mixture I.

Key to Coded and Transformed Data.

$$X_0 = \log \log x_0 \qquad\qquad X_2 = x_2$$

$$X_1 = \log 1000\, x_1 \qquad\qquad X_3 = x_2 \log 1000\, x_1$$

		X_0	X_1	X_2	X_3
		.41348	.301	40	12.04
		.29515	.301	140	42.14
		.48193	2.301	40	92.04
		.30472	2.301	140	322.14
Σ u		1.49528	5.204	360	468.36
	$\bar{u}$	.37382	1.301	90	117.09
Σ uv	X_0	.58319004	2.023379	119.7982	159.93526
	X_1		10.7704	468.36	969.33636
	X_2			42400.	55162.04
	X_3^2				114,166.2824
$\dfrac{\Sigma u \Sigma v}{n}$	X_0	.558965	1.945359	134.5752	175.08234
	X_1		6.770404	468.36	609.33636
	X_2			32400.	42152.40
	X_3^2				54840.272
SP(u,v)	X_0	.024225	.07802	-14.777	-15.14708
	X_1		4.0000	0	360.00
	X_2			10000.	13010.0
	X_3^2				59326.01

This gives the following three equations and three unknowns:

$$4\, b_1 + \quad 0\, b_2 + \quad 360 \quad b_3 = 0.07802$$

$$0\, b_1 + 10,000\, b_2 + 13,010 \quad b_3 = -14.777$$

$$360\, b_1 + 13,010\, b_2 + 59,326.01\, b_3 = -15.14708$$

Solving these equations, one obtains: $b_1 = 0.046027$, $b_2 = -0.001094$, $b_3 = -0.0002263$.

This gives: $X_0 = 0.37382 + 0.0460277 (X_1 - 1.301) - 0.0010941 (X_2 - 90) - 0.000294697 (X_3 - 117.09)$.

Collecting terms: $X_0 = 0.44515 + 0.0460277 X_1 - 0.0010941 X_2 - 0.000294697 X_3$.

Substituting in this equation, the maximum confined compressive stress is predicted for the following test conditions:

$40°F$	0.02 in./min. =	620 psi
	(0.002 in./min. =	155 psi
$100°F$	(0.02 in./min. =	189 psi
	(0.2 in./min. =	232 psi
$140°F$	0.02 in./min. =	97 psi

Confined Compression Test, Mixture II - 30 psi

All that was desired in this study was to obtain the raw data prediction equation from a limited number of results. The calculations and procedures are the same as for Mixture I.

Key to Coded and Transformed Data.

$$X_0 = \log \log x_0 \qquad X_2 = x_2$$

$$X_1 = \log 1000 \, x_1 \qquad X_3 = x_2 \log 1000 \, x_1$$

		X_0	X_1	X_2	X_3
		.42518	.301	40	12.04
		.34157	.301	140	42.14
		.48308	2.301	40	92.04
		.34988	2.301	140	322.14
Σu		1.59971	5.204	360	468.36
$\bar{u}$		.39993	1.301	90	117.09
Σuv	X_0	.653204	2.147432	133.1334	176.68595
	X_1		10.7704	468.36	969.33636
	X_2			42400.	55162.40
	X_3				114,166.2824
$\dfrac{\Sigma u \Sigma v}{n}$	X_0	.63977	2.081210	143.9739	187.31004
	X_1		6.770404	468.36	609.33636
	X_2			32400.	42152.40
	X_3				54840.272
$SP(u,v)$	X_0	.013434	.066222	-10.8405	-10.62409
	X_1		4.0000	0	360.000
	X_2			10000.	13010.
	X_3				59326.01

This gives the following three equations and three unknowns:

$$4 \, b_1 + \quad 0 \, b_2 + \quad 360 \quad b_3 = 0.066222$$

$$0 \, b_1 + 10{,}000 \, b_2 + 13{,}010 \quad b_3 = -10.8405$$

$$360 \, b_1 + 13{,}010 \, b_2 + 59{,}326.01 \, b_3 = -10.62409$$

Solving these equations, one obtains: b_1 = 0.0388725, b_2 = -0.0007614,
b_3 = -0.00024795.

This gives: X_0 = 0.39993 + 0.0388725 (X_1 - 1.301) - 0.0007614 (X_2 - 90) -
    0.00024795 (X_3 - 117.09).

Collecting terms: X_0 = 0.44542 + 0.0388725 X_1 - 0.0007614 X_2 -
     0.00024795 X_3.

  Substituting in this equation, the maximum confined compressive
stress is predicted for the following test conditions:

40°F	0.02 in./min. =	684 psi
	(0.002 in./min. =	230 psi
100°F	(0.02 in./min. =	276 psi
	(0.2 in./min. =	332 psi
140°F	0.02 in./min. =	162 psi

Unconfined, Repeated Load Test, Mixture II

From plots of results shown in Figures 34, 35, and 36 it was felt
that an equation representing that family of curves would have the
general form:

$$X_c = \left[E \cdot 10^{-\alpha(n-1)^B} + (1 - E) \right] X_0 \quad . \; . \; . \; . \; (1)$$

where X_c = applied, cycled, unconfined compressive stress, psi

X_0 = maximum unconfined compressive stress for the test

condition (fixed rate of deformation and temperature)

n = number of load repetitions necessary to cause excessive

shear deformations

E, α, and B = parameters that are dependent upon mixture composition.

From the data taken, E was approximated as 25%. This left α and
B to be evaluated. Equation 1 was broken down as follows:

$$\frac{\dfrac{X_c}{X_0} - .25}{.75} = 10^{-\alpha(n-1)^B}$$

if:

$$\frac{\dfrac{X_c}{X_0} - .25}{.75} = Y$$

then:

$$\log Y = -\alpha(n-1)^B$$

and

$$\log \log Y = B \log (n-1) - \log \alpha .$$

Now let $\log \log Y = Z$; $\log (n-1) = t$; $\log \alpha = b_1$; and $B = b_2$.
This gives: $Z = b_1 + b_2 t$.

The problem was to use a least squares analysis to solve for the
best values of b_1 and b_2.

Coded and Transformed Data.

t	z
.301	− .754
.699	− .321
.477	− .599
.778	− .321
.301	− .754
.778	− .321
.602	− .480
.778	− .321
.477	− .480
.778	− .321
.602	− .754
.778	− .321
0	− .480
.602	− .321
.301	− .754
.477	− .321
.477	− .754
.778	− .321

$t^2 = 6.38944$

$2zt = - 4.36724$

$z^2 = 4.81995$

$t = 9.9840$

$z = -8.698$

number of observations $= 18$

This gives the following two equations and two unknowns:

$$18\ b_1 + 9.984\ b_2 = -8.698$$

$$9.984\ b_1 + 6.38944\ b_2 = -4.367237$$

$$b_2 = 0.537$$

$$b_1 = -0.78103 \text{ or } \log \alpha = -0.78103 \text{ and } \alpha = 0.1656$$

The final form of the equation is:

$$X_c = \left[0.75 \cdot 10^{-0.1656(n - 1)^{0.537}} + 0.25 \right] X_0.$$